Abstract—The torque-maximizing field-weakening control scheme proposed by Kim and Sul is developed further. The performance under imperfect field orientation conditions is investigated, and it is shown that an overestimated—rather than an underestimated—model leakage inductance should be used. A slightly modified algorithm, which offers better robustness and reduced computational complexity, is presented. The importance, for good performance, of combining the scheme with current and speed controllers featuring antwindup and improved disturbance rejection is emphasized. The dynamics of the resulting closed-loop system are analyzed. Obtained in the process are rules for selection of all controller parameters, allowing tuning without trial-and-error steps. Good performance of the resulting system is verified experimentally.

Index Terms—Current control, field weakening, induction motor, speed control.

I. INTRODUCTION

FIELD-WEAKENING (FW) is made in order to allow operation of variable-speed induction motor (IM) drives at high speeds. The back electromotive force (EMF), electrical rotor speed times rotor flux ($\omega_r\Phi_r$), can become no larger than the available inverter voltage. Thus, to allow the speed to increase, the flux must be reduced.

The 1990s showed renewed interest in methods for FW control. For rotor-flux orientation, it was shown that the classical FW method of selecting the flux-producing current component $i_d$, proportional to $1/\omega_r$ yields too large a value; maximum torque is not achieved [1], [2]. Various strategies for improved FW operation were proposed [3]–[6].

Of particular interest, regarding ease of implementation and low parameter sensitivity, among the new schemes is that suggested by Kim and Sul [4]. (A related scheme was proposed by Grotstollen and Wiesling [3].) Starting from a synchronous-frame proportional–integral (PI)-type current controller, the basic idea is to select the reference (setpoint) $i_d^{\text{ref}}$ such that the modulus of the stator voltage vector $\mathbf{v} = v_d + jv_q$, $|\mathbf{v}| = \sqrt{v_d^2 + v_q^2}$, always equals $V_{\text{max}}$, the latter given by the inverter’s dc-link voltage. Thus, if $|\mathbf{v}| > V_{\text{max}}$ $i_d^{\text{ref}}$ is decreased, whereas if $|\mathbf{v}| < V_{\text{max}}$ $i_d^{\text{ref}}$ is increased, but, of course, only up to the value which yields nominal flux (at this point, the FW range is exited). Adjustment of $i_d^{\text{ref}}$ is made by a PI controller, whose input is $V_{\text{max}} - \sqrt{v_d^2 + v_q^2}$. As shown in [2], $|v_d| = |v_q| = V_{\text{max}}/\sqrt{2}$ yields maximum torque in the steady state (see also Section III). The strategy of [4] is, therefore, to adjust the torque-producing current reference $i_d^{\text{ref}}$ by another PI controller, whose input signal is $V_{\text{max}}/\sqrt{2} - |v_d|$

A variant of the FW scheme was applied, by the same authors, to permanent-magnet synchronous motors [7], and a similar strategy was independently developed by Maric et al. [8], [9]. However, while also in the latter two papers a PI controller is used for adjustment of $i_d^{\text{ref}}$, the controller input signal is $V_{\text{max}}^2 - v_d^2 - v_q^2$. This is an improvement, as square-root computation is avoided when implementing the algorithm. The choice is, therefore, adopted here.

The contributions of the present paper, which extends the results of [4], are as follows.

1) The importance of preventing integrator windup (which, if allowed to occur, results in deteriorating performance) is emphasized. So is controller design for good disturbance rejection. Some recent results are brought together in Section III within the framework of the FW control system, namely, current and speed controllers using “back-calculation” [10] (also known as “realizable references” [6]) and “active damping” [11]. Convenient rules for controller parameter selection are presented.

2) The performance under imperfect field orientation conditions is investigated. It is shown that, for good performance, the model leakage inductance should be overestimated.

3) The original scheme is slightly modified, also in Section III, with the $i_d^{\text{ref}}$-adjusting PI controller reduced to an integrator only, and the $i_q^{\text{ref}}$-adjusting controller replaced by a static relation between $i_d^{\text{ref}}$ and $i_q^{\text{ref}}$. The former reduces the complexity and the number of controller parameters to be selected, while the latter is made in order to tackle imperfect field orientation.

4) The dynamic performance of the resulting closed-loop system is analyzed in Section IV. Besides insight, this yields a convenient rule for selection of the gain of the $i_d^{\text{ref}}$-adjusting integrator, thus completely eliminating trial-and-error steps in the design and tuning procedure.

II. IM MODEL

An IM model with the leakage referred to the stator side (T-equivalent) and using rotor flux orientation is considered. The following symbols are used:

$$\mathbf{v} = v_d + jv_q$$

synchronous-frame stator voltage vector;
The space vectors are assumed to be rms-value scaled (the three-phase/two-phase scaling constant is selected as $1/\sqrt{3}$). In synchronous coordinates, the electrical part of the IM model is given by

$$
\mathbf{v} = (R_s + j\omega_1 L_{\sigma})\mathbf{i} + L_{\sigma}\frac{d\hat{\mathbf{q}}}{dt} + j\omega_1 \Psi + \frac{d\Psi}{dt}
$$

where

$$
\frac{d\Psi}{dt} = R_s \mathbf{i} - \left( R_{\sigma} \frac{L_{\sigma}}{L_M} + j(\omega_1 - \omega_p) \right) \Psi.
$$

Hence, in component form, we have

$$
L_{\sigma} \frac{d\mathbf{i}_d}{dt} = v_d - R_s \mathbf{i} + \omega_1 L_{\sigma} \mathbf{i}_d + \frac{R_{\sigma}}{L_{\sigma}} \Psi_d + \omega_1 \Psi_q
$$

$$
L_{\sigma} \frac{d\mathbf{i}_q}{dt} = v_q - R_s \mathbf{i} - \omega_1 L_{\sigma} \mathbf{i}_d + \frac{R_{\sigma}}{L_{\sigma}} \Psi_d - \omega_1 \Psi_q
$$

$$
\frac{d\mathbf{q}_d}{dt} = R_{\sigma} \mathbf{i} - \frac{R_{\sigma} L_{\sigma}}{L_M} \mathbf{i}_d - (\omega_1 - \omega_p) \mathbf{q}_d
$$

$$
\frac{d\mathbf{q}_q}{dt} = R_{\sigma} \mathbf{i} + \frac{R_{\sigma} L_{\sigma}}{L_M} \mathbf{i}_d - (\omega_1 + \omega_p) \mathbf{q}_d
$$

$$
\frac{J}{n_p} \frac{d\omega_p}{dt} = 3n_p (\Psi_d \mathbf{i}_q - \Psi_q \mathbf{i}_d) - \tau_L.
$$

Fast and accurate current control is assumed (see below), so the dynamics of (3) and (4) can be neglected as seen from the flux and speed dynamics, (5)–(7). In the following, a “hat,” e.g., $\hat{\mathbf{L}}_{\sigma}$, will denote a model machine parameter, i.e., the value used in the control algorithm, and a “tilde,” e.g., $\tilde{\mathbf{L}}_{\sigma}$, a parameter error.

The synchronous coordinate system is fixed to the estimated flux, $\hat{\mathbf{L}}_{\sigma}$ (where superscript $s$ denotes stator coordinates). Hence, $\hat{\mathbf{L}}_{\sigma}$ is real valued. If the field orientation is perfect, $\Psi$ is also real valued. This, however, cannot be taken for granted. At high speeds, the “voltage model” [12]

$$
\frac{d\hat{\mathbf{L}}_{\sigma}}{dt} = \mathbf{v}^* - \hat{\mathbf{R}}_s \hat{\mathbf{i}} - \hat{\mathbf{L}}_{\sigma} \frac{d\hat{\mathbf{L}}_{\sigma}}{dt} - \alpha_v \hat{\Psi}^*
$$

(or a variant thereof) is the desired flux estimator, due to its low parameter sensitivity. (The term $\alpha_v \hat{\Psi}^*$ is used for stability reasons, giving a low-pass integrator.) Neglecting $\alpha_v$ and the stator resistance, (8) combined with (1) yields the steady-state relation $\mathbf{v} = \hat{\mathbf{L}}_{\sigma} \tilde{\mathbf{i}}_d$, so

$$
\Psi_q = -\hat{\mathbf{L}}_{\sigma} \tilde{\mathbf{i}}_q.
$$

By combining this relation with (5) and (6), it is found (when approximating $1 + L_{\sigma}/L_M \approx 1$) that, in the steady state,

$$
\Psi_d = L_M \left( \frac{\mathbf{i}_d}{2} + \sqrt{\frac{\gamma_1^2}{4} - 2 \hat{\mathbf{L}}_{\sigma} \tilde{\mathbf{i}}_q^2} \right).
$$

Under imperfect field orientation, not only $\mathbf{i}_d$, but also to a degree $\tilde{\mathbf{i}}_q$, controls the flux level. In the next section, it will be seen that this has a significant influence on the system performance in the FW range, even for a small $\tilde{\mathbf{L}}_{\sigma}$. As they will be found useful later on, let us already at this stage calculate the steady-state voltage–current relations obtained by neglecting $R_s$ and substituting (9) and (10) into (1)

$$
v_d = -\omega_1 \hat{\mathbf{L}}_{\sigma} \xi \mathbf{i}_d
$$

$$
v_q = \omega_1 \left( \frac{L_{\sigma}}{L_M} + \frac{1}{2} + \sqrt{\frac{\xi^2 \hat{\mathbf{L}}_{\sigma}}{L_M}} \right) L_M \mathbf{i}_d
$$

where $\xi = \mathbf{i}_q / \mathbf{i}_d$.

### III. CONTROL SYSTEM DESIGN

#### A. Current Controller

Synchronous-frame current control, based on the results of [6], [10], and [11], is proposed. For clarity of the presentation, some key results of these papers are summarized.

The dynamics of (3) and (4) are governed by a complex pole pair at $-R_{\sigma}/L_{\sigma} \pm j\omega_1$. The dynamics are decoupled by an inner feedback loop, formed by selecting $\mathbf{v} = \mathbf{v}' + j\tilde{\omega}_1 \tilde{\mathbf{L}}_p \mathbf{1}$, where

$$
\mathbf{v}' = \mathbf{v}_d + j\mathbf{v}_q',
$$

thereby moving both poles to $-R_{\sigma}/L_{\sigma}$. The outer loop is then closed by two PI controllers, whose outputs are $\mathbf{v}_d$ and $\mathbf{v}_q'$, respectively. In complex notation, with $\mathbf{e} = \mathbf{i}^\mathbf{r} - \mathbf{i}^\mathbf{s}$

$$
\mathbf{v}' = \mathbf{K}_c (\mathbf{e} + \mathbf{1}),
$$

$$
\frac{d}{dt} \mathbf{e} = \frac{1}{T_{ic}} \mathbf{e} \Leftrightarrow \mathbf{v}' = \mathbf{K}_c \left( 1 + \frac{1}{p T_{ic}} \right) \mathbf{e}
$$

where $\mathbf{I} = \mathbf{I}_d + j\mathbf{I}_q$ is the complex integrator state variable and $p = d/dt$. By selecting $\mathbf{K}_c = \omega_1 \mathbf{L}_{\sigma}^c$ and $T_{ic} = \tilde{\mathbf{L}}_{\sigma}/\tilde{R}_c$, the poles of the decoupled system are, ideally, canceled, and the closed-loop transfer operator $\mathbf{K}_c/(p + \alpha_c)$ is obtained [10]. The bandwidth $\alpha_c$ is related to the 10%–90% rise time $T_{ic}$ as $\alpha_c T_{ic} = \frac{\gamma_1}{9} \approx 2.2$. Hence, a specification for the rise time immediately yields the bandwidth and, in turn, the appropriate controller parameters. However, as shown in [11], load disturbances (in form of a varying back EMF) are rejected with the canceled dynamics, $R_{\sigma}/L_{\sigma}$. This is, typically, a small value as compared to $\omega_1$. The disturbance rejection can be improved by introducing an “active damping” in the decoupling loop

$$
\mathbf{v} = \mathbf{K}_c (\mathbf{e} + \mathbf{1}) + (j\omega_1 \tilde{\mathbf{L}}_{\sigma} - \tilde{R}_c) \mathbf{i}
$$

where $\tilde{R}_c$ in [11] is called the “active resistance.” Now, if $\tilde{R}_c = \alpha_c \tilde{\mathbf{L}}_{\sigma} - \tilde{R} \approx \alpha_c \tilde{\mathbf{L}}_{\sigma}$ (the approximation since $\tilde{R}$ is small), the poles of the decoupled system are moved to $-\alpha_c$, and the disturbance rejection dynamics become as fast as the closed-loop dynamics. The integral action now has to be increased to cancel the
pole at $-R_a/L_\sigma = -\alpha_c$, so the controller parameters should be selected as

$$R_a = \alpha_c L_\sigma - \dot{R} \approx \alpha_c \dot{L}_\sigma = K_c \quad T_{ic} = \frac{1}{\alpha_c},$$  \hspace{1cm} (14)

We here assume that, in order to avoid low-order harmonics with increased torque ripple as a result, the pulsewidth modulation (PWM) is not extended all the way into the six-step mode in the steady state. Hence, $V_{\text{max}}$ is the radius of a circle inscribed in the space-vector modulation hexagon [12] (see Fig. 1). However, unless $\hat{v}_d^{\text{ref}}$ is decreased very quickly, $|v|$ will exceed $V_{\text{max}}$ at transients, i.e., the overmodulation range is entered. Here, though, there is little reason not to apply the maximum available voltage. The realizable vector $v = PWM(v)$ can be selected according to various methods. For details, see [13] and the references cited therein. The “minimum phase error” method—illustrated in Fig. 1—is straightforward to implement using digital PWM techniques.

In order to prevent integrator windup (resulting in deteriorating performance) in the overmodulation range, the integrators of the PI controllers should be updated using a modified error, $\varepsilon$. This is called the “realizable references” [6] or “back-calculation” [10] method. The “back-calculated” error $\varepsilon$ is that which, substituted into (13), would yield $V$. Thus, by comparing the calculation of the ideal vector: $v = K_e (\varepsilon + \dot{\varepsilon}) + (\dot{\omega}_1 L_\sigma - R_a) \dot{j}$ to that yielding $V$:

$$\varepsilon = e + \frac{1}{K_e} (v - \dot{v}) \Rightarrow \frac{\text{d} \varepsilon}{\text{d} t} = \frac{1}{T_{ic}} \left( e + \frac{1}{K_e} (\dot{v} - v) \right).$$  \hspace{1cm} (15)

A block diagram and the complete control algorithm are given in Section III-D.

For discrete-time implementation, the forward Euler method [14]—being the least computationally costly one—is suggested. That is, the integration $\text{d}L/\text{d}t = c$ is approximated as $L_{k+1} = L_k + T \cdot c$, where $T$ is the sample period and $k$ is the sample number (at time $t = kT$). Following [10], the angular sampling frequency $\omega_s$ should be selected as $\omega_s \geq 10 \alpha_c$, i.e., at least a decade above the closed-loop bandwidth.

To illustrate the benefit of using “active damping” combined with “back-calculation” (see also [11, Fig. 15]), current control of an IM with the per-unit parameter values $L_M = 1.75\Omega$, $L_\sigma = 0.25$, $R_s = 0.05$, $R_R = 0.03$, $\alpha_c = 5$, and a base speed of 314 rad/s (thus, giving an ideal rise time of 1.5 ms) is simulated [see Fig. 2(a)]. The sampling frequency is, according to the above rule, selected as $10 \alpha_c/2\pi = 2.5$ kHz, i.e., the sample period is 0.4 ms.

When $\omega_s \approx 0.8$ p.u. (but still varying slightly), $\hat{v}_d^{\text{ref}}$ is stepped to 0.1 p.u. at $t = 5$ ms and to 0.9 p.u. at $t = 10$ ms. The first step causes no voltage saturation, so the ideal rise time is obtained. At the second step, the overmodulation range is entered, and it is seen that when the “back-calculation” is removed, an overshoot occurs due to integrator windup. Removal of the “active damping” prevents $\varepsilon_d$ from fully reaching $\hat{v}_d^{\text{ref}}$ during the displayed time interval. This is due to the poorly rejected load disturbance (itself due to the varying speed). Although $\varepsilon_d$ eventually does converge to $\hat{v}_d^{\text{ref}}$—thanks to integral control action—the settling time is quite long.

Also, the robustness against computational and/or inverter switching time delays is studied. Simulations of delays of 1/2 and a full sample period are shown in Fig. 2(b). While the performance in the first case is adequate, in the latter case there is some undesirable ringing. This result agrees with that found in [15]. Hence, if the “$\omega_s = 10 \alpha_c$” rule is followed, it is important that the total time delay is shorter than the sample period. If this is not achievable, $\alpha_c$ should be lowered accordingly.

B. Reference Selection for Torque Maximization

The FW strategy in [4] is to adjust $\hat{v}_d^{\text{ref}}$ by a PI controller whose input is $V_{\text{max}} - \sqrt{\dot{v}_d^{2} + \dot{v}_q^{2}}$. A drawback of using a PI controller, however, is that an algebraic loop is created. Due to the $P$ term of the current controller—cf. (13)—$v_d$ is a direct function of $\hat{v}_d^{\text{ref}}$. Similarly, the $P$ term of the reference-adjusting controller causes $\hat{v}_d^{\text{ref}}$ to become a direct function of $v_d$ and, in turn, itself. This problem is resolved by using integration only

$$\frac{\text{d} \hat{v}_d^{\text{ref}}}{\text{d} t} = \gamma (V_{\text{max}} - \sqrt{\dot{v}_d^{2} + \dot{v}_q^{2}}), \quad \hat{v}_d^{\text{ref}} < \hat{v}_d^{\text{ref}} \leq \frac{\dot{v}_d}{L_M}$$  \hspace{1cm} (16)
where $\psi_{q,\text{min}}^\text{ref}$ is the minimum allowed value (to prevent complete demagnetization and $i_d$ from becoming negative at transients). This should yet be sufficiently small to allow adequate high-speed operation, e.g., $\psi_{\text{nom}}/10L_M$.

Regarding selection of $\psi_{q,\text{ref}}^\text{ref}$, it follows from (11) that if the field orientation can be assumed perfect, $L_\sigma = L_\sigma$, then

$$v_d = -\omega_1 L_\sigma i_q, \quad v_q = \omega_1 (L_\sigma + L_M) i_d. \quad (17)$$

The steady-state electrical torque is proportional to $i_d i_q$. From (17), it is found that when maximum voltage is applied, $v = V_{\text{max}} e^{j\phi}$, $i_d i_q$ is proportional to $v_d v_q = V_{\text{max}}^2 \cos 2\phi$. Thus, the torque is maximized if $\cos 2\phi = 1$, i.e., $|v_d| = |v_q| = V_{\text{max}}/\sqrt{2}$, giving $|v_q| = (L_\sigma + L_M) i_d/L_\sigma$ (see also [2]). So, if the model parameters are accurate, maximum torque can be obtained by selecting the references as

$$|\psi_{q,\text{ref}}^\text{ref}| = \hat{\xi}^\text{ref} i_d, \quad \hat{\xi} = \frac{\hat{L}_\sigma + \hat{L}_M}{L_\sigma}. \quad (18)$$

To circumvent the inherent parameter sensitivity of (18), the solution in [4] is to let $v_{\text{dmax}}/\sqrt{2} - |v_d|$ be the input to another PI controller, which adjusts $\psi_{q,\text{ref}}^\text{ref}$.

This strategy is, however, associated with two problems.

First, simulations show that a marginally stable system is often obtained. Oscillations tend to appear, particularly when maximum current and voltage is applied. This is perhaps seen also in [4, Fig. 8]: note the increased ripple in $\psi_{d,\text{ref}}^\text{ref}$ between times 0.4–1.4 s. While the oscillations are reduced by using "back-calculation," this instead tends to give hangup at erroneous values of $i_d$ and $i_q$.

Second, and more seriously, the "$|v_d| = |v_q|$ rule holds only for perfect field orientation. When $v_d^2 + v_q^2 = V_{\text{max}}^2$, it follows from (9) to (11) that the electrical torque can be expressed as

$$f(\xi) = \frac{\xi (\hat{\eta} + \frac{1}{2} + \sqrt{\frac{1}{4} - \xi^2 \hat{\eta}})}{(\xi \hat{\eta})^2 + \left(\eta + \frac{1}{2} + \sqrt{\frac{1}{4} - \xi^2 \hat{\eta}}\right)^2}. \quad (19)$$

where

$$\begin{align*}
\eta &= L_\sigma/L_M \\
\hat{\eta} &= \hat{L}_\sigma/L_M \\
\hat{L}_\sigma &= L_\sigma/L_M
\end{align*}$$

and, again,

$$\xi = i_q/i_d.$$ 

Similarly, we obtain

$$\frac{v_q}{v_d} = \frac{\eta + \frac{1}{2} + \sqrt{\frac{1}{4} - \xi^2 \hat{\eta}}}. \quad (20)$$

Fig. 3 shows (19) and (20) for $L_M = 2$ and $L_\sigma = 0.2$. When $\hat{L}_\sigma$ is accurate, the torque maximum occurs, as predicted, for $\xi = (L_\sigma + L_M)/L_\sigma = 11$, where also $|v_q|/v_d = 1$. When $\hat{L}_\sigma$ is overestimated (dashed curves), the torque is again maximized when $|v_q|/v_d = 1$, although this now occurs for an unreasonably large $\xi$ (beyond the range of the $\xi$ axis). Thus, the strategy of [4] may work, albeit with a very small $i_d$ as a result. On the contrary, even for a slightly underestimated $L_\sigma$, no $\xi$ exists for which $|v_q|/v_d = 1$ (in Fig. 3, $f(\xi)$ becomes complex for $\xi > 4.8$). Hence, there is no stable operating point. If, instead, (18) is used, the selection of $\hat{\xi}$ becomes critical. If too small, much less than maximum torque is produced; if too large (in Fig. 3 larger than 4.8), there again is no stable operating point.

As $f(\xi)$ flattens out for large $\xi$ when $\hat{L}_\sigma$ is overestimated, it is not essential to use the exact $\xi$ for which the torque is maximized, only a value sufficiently large. The following alternative strategy for selection of $\psi_{q,\text{ref}}^\text{ref}$ is then logical.

Step 1) Determine the likely maximum and minimum values of $L_\sigma$. These can be found from locked-rotor tests using rated current, giving the saturated inductance $L_\sigma^{\text{min}}$, and a current much smaller than the rated, giving $L_\sigma^{\text{max}}$. Also determine $L_M^{\text{max}}$ from a no-load test using a current smaller than the rated.

Step 2) Select $\hat{L}_\sigma = L_\sigma^{\text{max}}$ to guarantee an overestimate, and let

$$\hat{\xi} \geq \frac{L_\sigma^{\text{max}} + L_M^{\text{max}}}{L_\sigma^{\text{min}}}. \quad (21)$$

As the motor or inverter rating sets a limit $L_\sigma^{\text{max}}$ on the permissible current modulus, $\psi_{q,\text{ref}}^\text{ref}$ may not grow so large as to make $|i| > L_\sigma^{\text{max}}$. The proposed strategy can, therefore, be used only in "FW region II" (highest speeds), where maximum voltage is used but not maximum current [1], [4]. Otherwise, $\psi_{q,\text{ref}}^\text{ref}$ must be restricted as $|\psi_{q,\text{ref}}^\text{ref}| = \sqrt{L_\sigma^{\text{max}} - (\psi_{q,\text{ref}}^\text{ref})^2}$. Thus, maximum current and voltage is applied in this case, i.e., in "FW region I" (high speeds). Clearly, if both this and (18) give values that exceed the nominal reference, $\psi_{q,\text{nom}}^\text{ref}$ (which, if closed-loop speed control is used, is the speed controller output, see Section III-C), we
should select \( \tilde{q}_{\text{ref}} = q_{\text{ref}, \text{nom}} \). This choice is used for operation at nominal speeds or if maximum torque is not called for in the FW range.

The controller thus always operates in one of the following three modes:

- **Mode A**, nominal torque: \( \tilde{q}_{\text{ref}} = q_{\text{ref}, \text{nom}} \)
- **Mode B**, maximum current: \( \tilde{q}_{\text{ref}} = \sqrt{P_{\text{max}}^2 - (i_d^\text{ref})^2} \)
- **Mode C**, maximum torque: \( i_{\text{q}, \text{nom}} = \xi_i^\text{ref} \).

### C. Speed Controller

In the speed control loop, load disturbances (in the form of load torque variations) are often more pronounced than in the current control loop. Quite a few control methods for improved disturbance rejection have been proposed, many based on load torque estimation [16]–[18]. For simplicity, we here suggest a strategy similar to that for current control. By introducing an “active damping,” \( \dot{i}_q = \dot{q} - B_q \dot{\omega}_r \), the pole of (7) (which is located close to the origin if the natural damping, i.e., the incremental load torque with respect to the mechanical rotor speed, \( b_n = 1/\eta_n \cdot d\tau_1/d\omega_r \), is small) is moved to the desired closed-loop bandwidth, \( \alpha_s \). The linearized dynamics are

\[
\frac{d\omega_r}{dt} = \frac{3n^2y_2}{J} (\dot{\omega}_r - B_q \omega_r) - b_n \omega_r \Rightarrow \omega_r = \frac{3n^2y_2}{J} \frac{1}{p + \alpha_s} \omega_r^* \tag{22}
\]

Hence, \( B_q = (\alpha_s J - b_n)/3n^2y_2 \), ideally. The loop is then closed with a PI controller, which cancels the pole at \( -\alpha_s \): \( \omega_r^* = K_s(1 + 1/pT_{is}) \omega_r - \omega_r \). With \( \alpha_s \) as the bandwidth at nominal speeds,

\[
K_s = \frac{\alpha_s J}{3n^2y_2 \psi_{\text{nom}}} B_q = \frac{\alpha_s J - b_n}{3n^2y_2 \psi_{\text{nom}}} T_{is} = \frac{1}{\alpha_s} \tag{23}
\]

To prevent integrator windup, “back-calculation” is useful also in this case (see [19] for a related scheme). The speed controller, with \( L_s \) as integrator state variable, is given in the below algorithm. (The resulting type of controller is also known as integral–proportional (IP) or two-degrees-of-freedom PI.)

**Remark:** If \( \alpha_s \) is selected such that \( \alpha_s J \gg b_n \), \( J \) is the only critical parameter for the speed controller design. The natural damping \( b_n \) depends on the load conditions and may vary. If “active damping” were not introduced, appropriate selection of \( T_{is} \) would be difficult: a controller tuned for well-damped operation (\( b_n \) large) will give large overshoots as \( b_n \) decreases.

### D. Control Algorithm

The following control algorithm is obtained:

\[
\frac{dL_s}{dt} = \frac{1}{T_{is}} \left( \omega_{\text{ref}} - \omega_r + \frac{1}{K_s} (\tilde{q}_{\text{ref}} - q_{\text{ref}, \text{nom}}) \right) \tag{24}
\]

\[
q_{\text{ref}, \text{nom}} = K_s (\omega_{\text{ref}} - \omega_r + I_s) = B_q \omega_r \tag{25}
\]

\[
\frac{dq_{\text{ref}}^\text{ref}}{dt} = \gamma \left( \sqrt{P_{\text{max}}^2 - (i_d^\text{ref})^2} \right) \quad \text{max} \leq \tilde{q}_{\text{ref}} \leq \psi_{\text{hom}} L_M \tag{26}
\]

\[
\tilde{q}_{\text{ref}} = \min \left\{ \frac{q_{\text{ref}}^\text{ref}}{\psi_{\text{hom}}} \frac{1}{L_M} \right\} \times \text{sign} \left( \frac{q_{\text{ref}}^\text{ref}}{\psi_{\text{hom}}} \right) \tag{27}
\]

**Fig. 4. Block diagram of the FW control system (excluding the speed controller). “CC” is the “back-calculation” current controller. (To avoid clutter, stator/synchronous coordinate transformations are not shown.)**

\[
\frac{dv_q}{dt} = \frac{1}{T_{ic}} \left( \tilde{v}_q - i_d + \frac{1}{K_c} (\tilde{v}_d - v_d) \right) \quad \text{(28)}
\]

\[
\frac{dv_d}{dt} = \frac{1}{T_{ic}} (\tilde{v}_q - i_q + \frac{1}{K_e} (\tilde{v}_q - v_q)) \quad \text{(29)}
\]

\[
v_q = K_c (\tilde{v}_q^\text{ref} - i_q + I_q) - \omega_1 \dot{\xi}_r i_q - R_d i_d \quad \text{(30)}
\]

\[
v_d = K_c (\tilde{v}_d^\text{ref} - i_d + I_d) + \omega_1 \dot{\xi}_r i_d - R_s i_q \quad \text{(31)}
\]

\[
\begin{align*}
\tilde{v}_q & = K_c (\tilde{v}_q^\text{ref} - i_q + I_q) + \omega_1 \dot{\xi}_r i_q - R_d i_d \\
\tilde{v}_d & = K_c (\tilde{v}_d^\text{ref} - i_d + I_d) + \omega_1 \dot{\xi}_r i_d - R_s i_q
\end{align*}
\]

Note the multiplication by the sign of \( \tilde{q}_{\text{ref}, \text{nom}} \) in (27), which is made to allow negative torque at braking and reverse rotation. A block diagram of the resulting control system is depicted in Fig. 4.

There is now only one parameter, \( \gamma \), left to select. A selection rule will follow as a result of the analysis made in Section IV.

### IV. CLOSED-LOOP DYNAMICS

In this section, the dynamics of the resulting closed-loop system are analyzed. For this purpose, accurate model parameters, \( \dot{L}_s = L_s \) and \( \dot{\xi} = \xi \), can be assumed, giving \( \psi_d = \psi \) and \( \psi_q = 0 \).

#### A. Flux Dynamics

It is well known that for steps in \( i_d \), the flux responds exponentially with the rate \( R_R/L_M \). This is a slow response; the rotor time constant \( L_M/R_R \) is typically, in the 100- ms range. Considering smaller speed changes in the FW range, which require insignificant transients in \( i_q \) (thus, also in \( v_d = -\omega_1 L_s i_q \) ), \( v_q = \sqrt{P_{\text{max}}^2 - (i_d^\text{ref})^2} \) can be approximated as constant. Then, with the dynamics of \( \tilde{q}_{\text{ref}}, i_d, \) and \( i_q \) neglected as seen from the slower \( \psi \), we have, from (4)

\[
v_q = \omega_1 \left( L_s i_d + \dot{v}_q \right) \Rightarrow i_d = \frac{1}{L_s} \left( \frac{v_q}{\omega_1} - \psi \right) \tag{33}
\]

This is an indirect feedback loop for \( \psi \), created by the \( i_d^\text{ref} \)-adjusting integrator (when active). Substituting (33) into (5), we obtain

\[
\frac{d\psi}{dt} = \frac{R_R}{L_s} \frac{v_q}{\omega_1} - R_R \left( \frac{1}{L_s} + \frac{1}{L_M} \right) \psi \tag{34}
\]
This represents a flux speedup of about one decade (since $L_\sigma$ is, characteristically, 1/10 of $L_\varphi$). As the speed cannot increase faster than the flux decreases, the maximum achievable speed dynamics are, thus, given by $R_\varphi/L_\varphi$. For an IM of medium power rating, we may, as an example, consider $R_\varphi = 0.5 \, \Omega$ and $L_\varphi = 10 \, \text{mH}$, giving $R_\varphi/L_\varphi = 50 \, \text{rad/s}$ and a rise time of $2.2/50 = 44 \, \text{ms}$. This is sufficient even for fast servo drives. Therefore, the dynamic properties for smaller speed changes in the FW range should be excellent. (Note that the analysis is not valid in the nominal-speed range, as $v_\varphi$ then varies.)

### B. Current and Reference Dynamics

As $v_\varphi^2 + v_\varphi^2 \approx V_{\text{max}}^2$, (26) can be linearized as follows:

$$
\frac{dv_{\text{ref}}}{dt} = -2\gamma \left[ v_\varphi (v_\varphi - v_\varphi) + v_\varphi (v_\varphi - v_\varphi) \right]
$$

where $v_\varphi$ and $v_\varphi$ are the nominal voltage components, around which the linearization is made. Furthermore, let us assume that $v_{\text{ref}} \approx v_\varphi$, $\omega_1 > 0$, and $v_{\text{ref}} = \xi v_{\text{ref}}$. The latter will, as we shall see, for analysis purposes account for all three modes, not only Mode A. Then, with (35) replacing (26), the system (3), (4), and (26)–(32) is linear. Collecting $v_{\text{ref}}$ and $v_{\text{ref}}$ in the state vector $x$, we have $x = Ax + input$, where the characteristic polynomial for $A$ is given by

$$
\det(sI - A) = (s + \alpha_\varphi) (s + \frac{R + R_\varphi}{L_\varphi})^2 p(s)
$$

where

$$
p(s) = s^2 + \alpha_\varphi [1 + 2\gamma L_\varphi (v_\varphi + \xi v_\varphi)] s + 2\alpha_\varphi \gamma [(R - \xi - 1 \omega_1 L_\varphi) v_\varphi + (\xi + 1 \omega_1 L_\varphi) v_\varphi].
$$

Since $R_\varphi$ is chosen such that $(R + R_\varphi)/L_\varphi = \alpha_\varphi$, there are always three poles located at $-\alpha_\varphi$. Suppose that $\gamma$ is small and that $R$ can be neglected. Then,

$$
p(s) \approx (s + \alpha_\varphi)q(s), \quad q(s) = s + 2\gamma L_\varphi (v_\varphi + \xi v_\varphi).
$$

Thus, there are four poles at $-\alpha_\varphi$, which account for the closed-loop current dynamics. For small changes in $v_{\text{ref}}$ (in Mode A), the dynamics of $i_\varphi$ in the FW range are as good as in the nominal-speed range. This is a result which was also acknowledged experimentally in [4]. The pole given by $q(s)$ accounts for the dynamics of (35), which we shall now analyze.

**Mode A:** Nominal torque, $\frac{v_{\text{ref}}}{\varphi_{\text{nom}}}$, is, for analysis purposes, equivalent to letting $\xi = 0$. As $\xi$ is small, $v_\varphi \approx V_{\text{max}}$ for $\omega_1 > 0$. Hence, $q(s) = s + 2\gamma L_\varphi V_{\text{max}}$. The dynamics of (35) should not be slower than the closed-loop flux dynamics, (34). Letting (35) be twice as fast as (34), i.e., letting $2\gamma L_\varphi V_{\text{max}} = 2R_\varphi/L_\varphi$, yields the parameter selection recommendation

$$
\gamma = \frac{R_\varphi}{\omega_0 L_\varphi^2 V_{\text{max}}}, \quad \omega_0 = \left\{ \begin{array}{ll}
\omega_{\text{base}}, & |\omega_1| \leq |\omega_{\text{base}}| \\
|\omega_1|, & |\omega_1| > |\omega_{\text{base}}|
\end{array} \right.
$$

where the modulus of $\omega_1$ results from a treatment similar to the above for $\omega_1 < 0$. Since $2R_\varphi/L_\varphi \ll \alpha_\varphi$, normally, the assumption that $\gamma$ is small should hold.

**Mode B:** Differentiating the relation $v_{\text{ref}}^2 + v_{\text{ref}}^2 \approx V_{\text{max}}^2$, the linearization $v_{\text{ref}} \approx 0$, and $v_{\text{ref}} = \xi v_{\text{ref}}$, about $\{v_\varphi, v_\varphi\}$ is obtained. Therefore, in this case we should let $\xi = i_\varphi/v_\varphi$, yielding

$$
-\xi v_\varphi + v_\varphi = \omega_1 \left( \frac{i_\varphi}{v_\varphi} (-L\sigma i_\varphi) + L\sigma i_\varphi + \psi \right) = \omega_1 \psi.
$$

Since $\psi \approx V_{\text{max}}/\omega_1$, this and (39) substituted into (38) yields

$$
p(s) = s + 2R_\varphi/L_\varphi.
$$

That is, the desired dynamics of (35) are obtained also in Mode B.

**Mode C:** Now, $\xi = (L_\sigma + L_\varphi)/L_\varphi$ and $-v_\varphi = v_{\text{ref}}/\sqrt{2}$. In this case, the dynamics of (35) are speeded up, and the factorization (38) is no longer valid. Considering instead (37), again with $R$ neglected and using (39), we have

$$
p(\lambda) = \lambda^2 + \left( 1 + \frac{\sqrt{2}R_\varphi L_\varphi}{\omega_1 L_\varphi} \right) \lambda + \frac{\sqrt{2}R_\varphi}{\alpha_\varphi L_\varphi} + \frac{L_\varphi}{L_\varphi}
$$

where $\lambda = s/\alpha_\varphi$. In order for the dynamics to be well damped, each root of $p(\lambda$) should have an imaginary part no larger than the real part (moduluswise). For a worst case scenario, suppose that $\omega_1$ is large. Suppose further that $L_\varphi/\alpha_\varphi L_\varphi = \sigma$. Then, for the imaginary part to be smaller than the real part, $12\sqrt{2} - 1 < 1/\alpha < 1/4 \Rightarrow \sigma < 1/24$, i.e., $\alpha_\varphi > 34R_\varphi/L_\varphi$. This recommendation yields reasonable bandwidths. In per-unit values, $R_\varphi/L_\varphi \approx 1/10$ typically, thus yielding $\alpha_\varphi = 3.4 \, \text{p.u.}$, which is smaller than the 5 p.u. used in simulations. Picking again $R_\varphi = 0.5 \, \Omega$ and $L_\varphi = 10 \, \text{mH}$, we get $\alpha_\varphi = 1700 \, \text{rad/s}$ and a current rise time of 1.3 ms.

### V. Evaluation

Experimental results for a four-pole 1.5-kW IM, connected to a dc-load machine, are presented. The IM has the following data: ELIN LKM-309, 220/380 V, 6.6/3.8 A, 1415 r/min, 50 Hz. The base values are: 3.8 A, 0.62 Wb, 195 V, 52 A, 0.16 H, and the parameters: $L_\varphi = 1.70$, $L_\varphi = 0.18$, $R_\varphi = 0.11$, $R_\varphi = 0.070$ p.u. The total inertia $J$ is approximately 700 p.u., and the load torque is roughly proportional to the rotor speed. The inverter uses a 5.3-kHz switching frequency and a 540-V dc-link voltage. The control system is implemented on a Texas Instruments TMS320C40 floating-point digital signal processor using a 5.3-kHz sampling frequency. Forward Euler discretization is used in all algorithms. The current and speed control loops are tuned for rise times of 1.4 and 140 ms, respectively ($\alpha_\varphi = 3.4 \, \text{p.u.}$), which is smaller than the 5 p.u. used in simulations. Picking again $R_\varphi = 0.5 \, \Omega$ and $L_\varphi = 10 \, \text{mH}$, we get $\alpha_\varphi = 1700 \, \text{rad/s}$ and a current rise time of 1.3 ms.
to 0.25 p.u.; see Fig. 6. As predicted by theory, fast response is obtained in the lower FW range, where maximum stator current is not required. As a diode rectifier is used, power cannot be fed back to the mains; hence, the slower response when the speed decreases. The ripple in $\hat{\gamma}_d$ in the experiments is the notable disagreement with the simulations. This can be explained as follows. Harmonics due to a nonsinusoidal air-gap flux are always present in ac machines. With closed-loop control—using a fairly high bandwidth—the stator current is regulated such that it becomes almost ideally sinusoidal. This, however, is at the expense of less ideal a stator voltage, giving, via $\hat{\gamma}_d + \gamma_d$, ripple in the $\hat{\gamma}_d$-adjusting integrator. Fortunately, this is attenuated by the slower flux dynamics and does not cause any significant torque ripple.

VI. CONCLUSION

The FW scheme in [4] was slightly modified, mainly in order to gain robustness to imperfect field orientation. The $\hat{\gamma}_d$-adjusting controller was replaced by the relation $\hat{\gamma}_d = \xi \hat{\gamma}_d$. For good performance, “back-calculation” and “active damping” were suggested for the current and speed controllers. The closed-loop dynamics were analyzed, yielding a design rule for the gain $\gamma$ of the $\hat{\gamma}_d$-adjusting integrator. The analytical expressions for the current and speed controller parameters, (14) and (23), and the selection rules for $\xi$ and $\gamma$, (21) and (39), should allow straightforward implementation and tuning of the control system.

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REFERENCES

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