User pairing schemes for capacity maximization in non-orthogonal multiple access systems
Muhammad Basit Shahab, Mohammad Irfan, Md Fazlul Kader and Soo Young Shin*
Wireless and Emerging Network System Lab, Kumoh National Institute of Technology, Gumi, South Korea

ABSTRACT
This paper addresses the issues related with conventional near–far user pairing in non-orthogonal multiple access. Performance effects of near–far pairing on regions with negligible channel gain differences between users are investigated. These regions occur when pairing is performed between cell center and cell edge users, thus leaving the cell mid users to be either paired with each other or kept unpaired. Pairing these mid users with each other causes successive interference cancelation (SIC) performance degradation resulting in capacity reduction for these users. On the other hand, leaving these mid users unpaired perfectly avoids the SIC issue but makes these users unable to benefit from the capacity gains provided by non-orthogonal multiple access. Therefore, two user pairing strategies have been proposed that can provide capacity gains to almost all the users by accommodating them in pairs, while avoiding or minimizing the mid users pairing problem. A generalized M-users pairing scheme is also proposed. Simulations have been performed to investigate the performance of proposed schemes for both perfect and imperfect SIC receiver scenarios in comparison with conventional pairing where the mid users are kept paired with each other. Simulation results show that proposed schemes achieve high capacity gains, especially when imperfect SIC is considered. Copyright © 2016 John Wiley & Sons, Ltd.

KEYWORDS
user pairing; capacity; non-orthogonal multiple access (NOMA); successive interference cancelation (SIC)

1. INTRODUCTION
The upswing and diversity in multimedia applications are transforming the nature of wireless data traffic by demanding high capacities and data rates. Furthermore, the expeditiously developing Internet of Things is increasing the volume of connected devices to an incredible extent. These humongous static and mobile devices will soon surpass the present number by a large magnitude. To fulfill the incredibly high user data rates and system capacity requirements, future radio access is aiming towards the design and implementation of 5G [1]. Among the key players in 5G, non-orthogonal multiple access (NOMA) is being considered as one of the appealing candidates to achieve manifold capacity gains because of its high spectral efficiency [2,3].

In NOMA, signals of multiple users are multiplexed in the power domain, thus being able to use the same frequency band. The multiplexed multiple users are said to be in a user pair. Impact of user pairing on the achievable sum rates of NOMA has been carried out in [4]. It is shown that users with more channel gain differences should be paired to ensure that NOMA sum rates are not less than conventional multiple access (MA) schemes. In [5], two schemes, namely, NOMA with fixed power allocation and cognitive inspired NOMA, are proposed to increase the capacity gap between NOMA and MA. A user pairing scheme based on the power requirements of users is given in [6]. Proportional fairness based user pairing and power allocation has been discussed in [7]. The works in [8–10] also focus on the user fairness issue and power allocation in NOMA.

Users in a cell are normally divided into three categories based on the channel quality indicator (CQI) values, namely, high quality, medium quality, and low quality users [11–15]. When we pair the high and low quality users with each other for better capacity gains, users of medium quality are left unpaired. If these medium quality users are paired with each other, the small channel gain difference between these in-pair users causes successive interference cancelation (SIC) performance degradation that results in capacity decrease for these users [16–18]. On the other hand, leaving these mid users unpaired perfectly avoids the SIC imperfection issue but makes these users unable to benefit from the capacity gains provided by NOMA.
best of our knowledge, user pairing schemes that consider the mid users pairing problem have not been proposed in the existing literature.

Moreover, in conventional near–far pairing, the capacity gain achieved by low-gain users is more than that of the high-gain users. This is because, for each user pair, the targeted data rate of low-gain user must be satisfied first, thus utilizing a high portion of power. The remaining less power is then allocated to the high-gain user. Because of this, the high-gain user cannot get enough benefit and has less capacity gain. Therefore, the overall capacity gain of the system decreases.

Principle contributions of this paper are summarized as follows:

- Firstly, we investigate the performance issues that arise when conventional near–far user pairing is performed. Region with negligible channel gain difference between in-pair users has been highlighted, and the corresponding effects on different performance metrics have been explored.
- Secondly, two schemes for users pairing are proposed by exploiting the trade-offs incorporated when pairing is performed. A generalized \( M \)-users pairing scheme is then developed, which divides cell users into multiple groups, followed by one of the proposed schemes for inter-group user pairing.
- We analytically derive the exact ergodic sum capacity of a two users pair considering the effects of perfect and imperfect SIC. The analytical results are later verified through simulations.
- Finally, we compare the achievable capacity of the proposed techniques with conventional near–far pairing by considering both perfect and imperfect SIC receivers. Effect of increase in the number of paired users is also investigated.

Rest of the paper is organized as follows. Section 2 explains the basic NOMA transmission protocol. Section 3 addresses the conventional near–far user pairing concept and the related issues. Proposed user pairing schemes are explained in Section 4. Mathematical analysis of the exact ergodic sum capacity of a two users pair considering perfect and imperfect SIC is provided in Section 5. Simulation results and discussions are carried out in Section 6. Finally, Section 7 concludes the paper.

2. NON-ORTHOGONAL MULTIPLE ACCESS

Consider a circular cellular region of radius \( R \) with the base station (BS) at the center serving a total of \( N \) users. Suppose \( M \) randomly distributed users out of \( N \) are multiplexed in the power domain. For any \( m \)-th user at distance \( d_m \) from the BS, its channel gain is represented by \( |h_m|^2 \), where \( |h_m|^2 \) represents the cell center user with highest gain. Let the user power allocation factor be represented by \( a_m \), and then, the powers allocated to these users are in the order \( a_1 \geq a_2 \cdots \geq a_M \). For any \( m \)-th user, the signal transmitted by the BS is \( \sqrt{a_m}p_m \), where \( p_m \) is the message signal for the \( m \)-th user and \( P \) is the total transmit power. So for \( M \) users in a pair, the cumulative signal sent by the BS is given by

\[
y = \sum_{m=1}^{M} \sqrt{a_m}p_m. \tag{1}
\]

The signal received at the \( m \)-th user can be written as

\[
r_m = h_m \sum_{m=1}^{M} \sqrt{a_m}p_m + \eta_m, \tag{2}
\]

where \( \eta_m \sim \mathcal{CN}(0, \sigma^2) \) represents the complex additive white Gaussian noise with mean zero and variance \( \sigma^2 \) of the \( m \)-th user. Because users sharing the same band will receive the signals of other users along with their own signals, interference becomes a problem. Thus, for all \( m \geq i \), the \( m \)-th user will have to detect and remove the data of all \( i \)-th users using SIC. On the other hand, each \( i \)-th user will treat the signals of all \( m \)-th users as noise. The total achievable data rate of \( M \) paired users can be calculated using

\[
R_{pair} = \sum_{m=1}^{M-1} \log_2 \left( 1 + \frac{a_m|h_m|^2}{\sum_{m=0}^{M-1} a_j + \frac{1}{\rho}} \right) + \log_2 \left( 1 + a_M|h_M|^2\rho \right), \tag{3}
\]

where \( \rho \) represents the transmit signal to noise ratio (SNR) per pair. For a cellular region with \( N \) users divided into \( L \) user pairs having \( M \) users per pair, the total achievable system capacity is given by

\[
R = \sum_{l=1}^{L} \sum_{m=1}^{M-1} \log_2 \left( 1 + \frac{a_m|h_m|^2}{\sum_{m=0}^{M-1} a_j + \frac{1}{\rho_l}} \right) + \log_2 \left( 1 + a_M|h_M|^2\rho_l \right), \tag{4}
\]

where \( \rho_l \) is the transmit SNR between \( M \) users of the \( l \)-th user pair. It is evident from (4) that the overall capacity of downlink NOMA depends majorly on the channel gains of users per pair and their allocated transmit powers. User pairing in NOMA is explained in the next section.

3. USER PAIRING IN NOMA

To analyze conventional near–far pairing in NOMA, we will restrict our discussion to the simplest case of two users per pair. To maintain maximum channel gain difference between in-pair users, it is preferred to combine users from the cell center (high CQI) and cell edge (low
CQI) into pairs. The cell mid users (medium CQI) thus left unpaired also need to be accommodated. If the cell mid users are paired with each other, the channel gain difference between these in-pair users is very less. The scenario has been depicted in Figure 1.

Considering inverse relation between channel gains and allocated powers [16,17].

Figure 1. Conventional near–far user pairing in non-orthogonal multiple access.

The probability $P\{R_i > \bar{R}_i\} = P\left(|h_i|^2 < \frac{1 - 2a_j^2}{\rho a_j^2}\right)$, (7)

$P\{R_j > \bar{R}_j\} = P\left(|h_j|^2 > \frac{1 - 2a_j^2}{\rho a_j^2}\right)$, (8)

These probabilities impose upper and lower bounds on the channel gains of low-gain and high-gain users in a pair, respectively. These probabilities will be high if the channel gain difference between in-pair users is kept more than a threshold. Therefore, if the cell mid users are paired with each other, their individual and cumulative data rates can be even less than MA.

Secondly, if cell mid users are still paired, their gain difference can become negligible because of little mobility. This causes in-pair users interference to be very high, requiring them to be un-paired and then re-paired with some other users. This leads to continuous un-pairing and re-pairing, thus increasing the computational complexity, signaling overhead, and time delays at the transmitter.

Furthermore, when near–far users are paired, the power allocated to low-gain user is very high compared with high-gain user. Consider three users $i$, $j$, $k$, such that $|h_i|^2 < |h_j|^2 < |h_k|^2$. For the low-gain user $i$, capacity comparison of user pairs $(i,j)$ and $(i,k)$ is given by (9)

$$\log_2 \left(1 + \frac{\rho |h_i|^2 a_{ijk}}{\rho |h_j|^2 a_{ij} + 1}\right)$$

$$\log_2 \left(1 + \frac{\rho |h_i|^2 a_{ijk}}{\rho |h_k|^2 a_{ij} + 1}\right),$$

given that $a_{ijk} > a_{ij}$ and $a_k < a_j$, where $a_{ijk}$ and $a_{ij}$ are power factors allocated to the $i$th user when paired with users $k$ and $j$, respectively. According to (9), capacity of low-gain user increases when paired with a more high-gain user. On the other hand, for the high-gain user $k$, its capacity gain decreases when paired with a more low-gain user because of getting low power. This is shown in (10)

$$\log_2 \left(1 + \frac{\rho |h_k|^2 a_{ij}}{\rho |h_i|^2 a_{ij}}\right)$$

$$\log_2 \left(1 + \frac{\rho |h_k|^2 a_{ij}}{\rho |h_j|^2 a_{ij}}\right),$$

given that $a_{ij} > a_{ij}$.

It is clear from (9) that low-gain users want to get paired with high-gain users. But from (10), it is evident that the reverse is not true. So, a trade-off needs to be required for pairing the users. It can be verified using (9) and (10) that the overall effect of such a pairing will be a decrease in the capacity gain of the pair. This is because when the power of high-gain user is decreased by a proportion, its capacity decreases by some factor. Contrary to this, when the power of low gain is increased by the same proportion, its capacity gain will be less than the capacity loss of high-gain user.
Although near–far pairing successfully achieves some capacity gains for the cell edge users, but the issues that arise for the middle and high-gain users are a big area of concern. Following section explains the proposed user pairing schemes.

4. PROPOSED PAIRING SCHEMES

Considering the highlighted points in Section 3, it is evident that a user pairing scheme should embed two considerations during pairing.

- Overall sum of in-pair users channel gain differences for all the pairs should be maximized.
- The channel gain difference between any two in-pair users should be greater than a threshold.

These two conditions are summarized as in (11)

\[
\max_{\forall_{i,j}} \sum_{l=1}^{L} |h_{il}|^2 - |h_{lj}|^2, \quad \text{subject to } \ |h_{il}|^2 - |h_{lj}|^2| \geq \gamma.
\]

where \(|h_{il}|^2\) and \(|h_{lj}|^2\) are gains of two users in the \(i_l\) pair and \(\gamma\) represents the minimum allowed channel gain difference between any two in-pair users considering the probabilities mentioned earlier in (7) and (8).

Any user pairing algorithm must satisfy the condition mentioned in (11). Now, consider the case of two users pairing. Let \(\bar{h}\) represent the median of channel gains of all the users to be paired. Then, if pairing is performed in a way that each pair contains one user with gain less than the median and other user having gain greater than the median, then the overall sum of in-pair users gain differences is constant as expressed in (12)

\[
\sum_{l=1}^{L} (|h_{il}|^2 - |h_{lj}|^2) = K.
\]

where \(|h_{il}|^2 > \bar{h}\) and \(|h_{lj}|^2 < \bar{h}\). Considering the issues related with conventional near–far pairing and condition mentioned in (11) in the light of (12), two methodologies for user pairing have been proposed.

4.1. Uniform channel gain difference pairing

Uniform channel gain difference (UCGD) pairing focuses on accommodating the cell mid users by maintaining a relatively UCGD between in-pair users of all pairs. Consider two users per pair case. Channel gains of users present in the cell are divided into two groups, followed by inter-group paring in such a way that cell mid users are well accommodated. Consider \(N\) users in a cellular area with sorted channel gains \(|h_1|^2, |h_2|^2, \ldots, |h_N|^2\). Users channel gains are sorted such that \(|h_1|^2 \leq |h_2|^2 \leq \cdots \leq |h_N|^2\). Groups are made as shown in (13) and (14)

\[
G_1 = \{ |h_i|^2 : |h_i|^2 < \bar{h} \}, \quad (13)
\]

\[
G_2 = \{ |h_i|^2 : |h_i|^2 > \bar{h} \}, \quad (14)
\]

User gains \(|h_1|^2, |h_2|^2, \ldots, |h_N|^2\) in group 1 are below the median \(\bar{h}\), while gains \(|h_{N+1}|^2, |h_{N+2}|^2, \cdots, |h_N|^2\) in group 2 are above it. Median is considered as a center point because it equally divides users into two groups. Considering uniformly distributed even number of users in a cell, \(\bar{h}\) will be an average of gains \(|h_{\frac{N}{2}}|^2\) and \(|h_{\frac{N}{2}+1}|^2\) as in (15)

\[
\bar{h} = \frac{|h_{\frac{N}{2}}|^2 + |h_{\frac{N}{2}+1}|^2}{2}. \quad (15)
\]

Two users (one from each group) can be paired if (16) is satisfied.

\[
||h_j|^2 - |h_i|^2| > \gamma \quad \forall i \in G_1, j \in G_2. \quad (16)
\]

For a user \(i\) from any group, the set \(G_{ji}\) of users \(j\) from the other group out of which any user can be paired with \(i\) is given by (17)

\[
G_{ji} = \{ j \mid (||h_j|^2 - |h_i|^2| > \gamma) \}. \quad (17)
\]

Consider the groups \(G_1\) and \(G_2\) defined above for two users pairing. In the proposed UCGD pairing, for any user \(i \in G_1\), it is paired with a user \(j \in G_2\) using (18)

\[
\text{Pair}_l = \{ i,j \mid i \in G_1, j = \min(G_{ji}) \}, \quad \forall \ 1 \leq l \leq L. \quad (18)
\]

Conversely, for any user \(i \in G_2\), it is paired with a user \(j \in G_1\) using (19)

\[
\text{Pair}_l = \{ i,j \mid i \in G_2, j = \max(G_{ji}) \}, \quad \forall \ 1 \leq l \leq L. \quad (19)
\]

It is obvious from (18) and (19) that UCGD pairing does not focus on pairing the cell center and cell edge users with each other. The technique actually finds users from the cell mid that can be paired with either the cell center or cell edge users. In this way, the proposed technique accommodates these mid users by efficiently pairing them with cell center and cell edge users, thereby allowing them to avail high capacity gains provided by NOMA and still avoiding or minimizing the SIC performance issue.

Consider a simplest case where large number of users are uniformly distributed in a cell and are divided into two groups according to (13) and (14). In this case, if there is one to one correspondence between users from two groups,
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This means that the minimum gain user of one group is paired with minimum of the other group. This process is continued until the highest gain users from both groups are paired. The final pairing results produced by UCGD pairing scheme as per (20) are shown in (21):

\[
\text{Pair}_1 = \left( \frac{|h_i|^2, |h_{\frac{N}{2}+i}|^2}{|h_j|^2, |h_{\frac{N}{2}+j}|^2} \right), \quad \forall 1 \leq i, j \leq \frac{N}{2}
\] (20)

From (21), it can be deduced that the average channel gain difference between in-pair users follows a comparably uniform behavior. Furthermore, when users in a cell increase, the pairing can more comfortably pair users because of one to one correspondence between user groups. After applying UCGD pairing, the scenario presented in Figure 1 can be expressed in Figure 2. This figure can give a basic idea of the UCGD pairing scheme for simple understanding.

Based on the conditions mentioned in (5)–(8), the capacity comparisons in (9) and (10) and Figure 2, capacity analysis of users from each category can be performed.

The major advantage of UCGD pairing is to accommodate cell mid users, causing a considerable capacity increase for these users. This is because now they are neither paired with less gain difference users from their own category nor left alone for using data with MA. Pairing mid users with other category users reduces the interference at the cell mid. This causes capacity increase in the cell mid, especially for imperfect SIC receivers.

Low-gain users data rates for UCGD will be slightly reduced if compared with conventional near–far pairing. This is because they are now paired with cell mid users instead of cell center, which require slightly more power compared with the cell center users. Thus, a slight reduction in the power allocation for the cell edge users is observed, reducing their data rates slightly.

Conversely, the data rates of high-gain users are higher for UCGD as compared with conventional near–far. This is because now the high-gain users are paired with cell mid users instead of cell edge users, whose power requirement is less than the cell edge users. This allows some increase in the power allocation for high-gain users, thus increasing their capacities.

4.2. Hybrid pairing

Hybrid user pairing scheme follows conventional near–far pairing for the extreme end users with high channel gain differences, but when the channel gain difference between users start to decrease, it switches to the UCGD pairing. This scheme continues to achieve the same data rates for extreme low-gain and high-gain users as in conventional pairing while still being able to minimize the mid users issue by making some trade-offs.

Consider the sorted user channel gains \(|h_1|^2, |h_2|^2, \ldots |h_k|^2, \ldots |h_{\frac{k}{2}+1}|^2, |h_{\frac{k}{2}+2}|^2, \ldots |h_{\frac{N}{2}}|^2, \ldots |h_N|^2, \ldots |h_{\frac{k}{2}+k}|^2, \ldots |h_N|^2\), where \(k\) indicates the point from where the pairing schemes are switched. Users with channel gains less and more than \(|h_k|^2\) and \(|h_{\frac{k}{2}+k}|^2\) respectively are paired using conventional near–far pairing, while the rest using UCGD pairing. Two users groups made according to (13) and (14) are shown below in (22) and (23).

\[
G_1 = \left\{ |h_1|^2, |h_2|^2, \ldots |h_k|^2, |h_{\frac{k}{2}+1}|^2, |h_{\frac{k}{2}+2}|^2, \ldots |h_{\frac{N}{2}}|^2 \right\}
\] (22)

\[
G_2 = \left\{ |h_{\frac{k}{2}+1}|^2, |h_{\frac{k}{2}+2}|^2, \ldots |h_{\frac{k}{2}+k}|^2, \ldots |h_N|^2 \right\}
\] (23)

For a user \(i\) from any group, the set of users \(j\) from the other group that can be paired with \(i\) is same as defined in (17). Considering the two groups \(G_1\) and \(G_2\) for two users pairing, any user \(i \in G_1\) can be paired with user \(j \in G_2\) according to (24)

\[
\text{Pair}_1 = \left\{ i, j \mid j = \max (G_{ji}), \quad \forall 1 \leq i < k \right\}
\]

\[
\text{Pair}_2 = \left\{ i, j \mid j = \min (G_{ji}), \quad \forall k < i \leq \frac{N}{2} \right\}
\] (24)

Conversely, for any user \(i \in G_2\), it is paired with user \(j \in G_1\) using (25)

\[
\text{Pair}_1 = \left\{ i, j \mid j = \max (G_{ji}), \quad \forall \frac{N}{2} < i < \frac{N}{2} + k \right\}
\]

\[
\text{Pair}_2 = \left\{ i, j \mid j = \min (G_{ji}), \quad \forall \frac{N}{2} + k < i \leq N \right\}
\] (25)

Consider the case where large number of users are uniformly distributed in a cell and are divided into two groups as defined in (22) and (23). In this case, for sorted users in
Pairing results of hybrid scheme are shown in (27)

\[
\text{Pair}_l = \begin{cases} 
|h_i|^2, |h_N|^2, & \forall 1 \leq i, l < k \\
|h_i|^2, |h_N|_{i+k+1}^2, & \forall k \leq i, l \leq \frac{N}{2} 
\end{cases}
\]  

Pairing results of hybrid scheme are shown in (27)

\[
\text{Pair}_l = \begin{pmatrix} 
|h_1|^2, |h_N|^2 \\
|h_2|^2, |h_N-1|^2 \\
... \\
|h_{k-1}|^2, |h_{N-(k-2)}|^2 \\
|h_k|^2, |h_{N-k}|^2 \\
|h_{k+1}|^2, |h_{N-k+1}|^2 \\
... \\
|h_{l}|^2, |h_{N-l}|^2 \\
\end{pmatrix} 
\]  

The result in (27) shows that initial pairs are made by joining the end users, for example, \(|h_1|^2\) and \(|h_N|^2\) being the lowest and highest gain users are paired, followed by pairing the next lowest \(|h_2|^2\) and highest \(|h_{N-1}|^2\), and so on. Once a specific point \(k\) is reached, then from this point further, pairs are made using UCGD pairing scheme. The scenario presented in Figure 1 can be expressed in Figure 3 after hybrid pairing is performed.

In comparison to Figure 1, it can be seen in Figure 3 that to accommodate the two near users UE4 and UE5, the two immediate next users (UE3 and UE6) will make a compromise. So, UE3 will be paired with UE6, and UE4 will be paired with UE6 using UCGD pairing. Thus, if some \(X\) number of users are very close to each other in the center and cannot be paired, then \(\frac{X}{2}\) users from both sides will accommodate them using UCGD pairing. Thus, the switching point \(k\) depends upon the number of near gain users in the center that cannot be paired.

This scheme achieves same capacities for extreme high and low-gain users as in conventional pairing. Though some of the high-gain and low-gain users that are not on the extremes are used to accommodate cell mid users. For these high-gain and low-gain users, the capacity analysis is same as was carried out earlier for UCGD pairing. High interference at the cell mid is still reduced here but with some compromises from those high-gain and low-gain users that are paired with them.

### 4.3. M-users UCGD pairing

We propose a generalized \(M\)-users pairing model by extending the concepts of UCGD pairing, where \(M\) represents the number of users in each pair. Consider \(N\) users in a cell with sorted channel gains. Let \(M\) represent the maximum number of users possible in a pair considering the minimum channel gain difference requirement of the in-pair users. We divide the cellular users into \(G\) groups, where \(G = M\). Because of a large number of users in a cell, we assume that the number of users in each group is the same.

For a user \(i\) from any group, it can be paired with a user \(j\) from the immediately next higher gain group using (28)

\[
\text{Pair}_l = \{i, j | j = \min(G_{ji})\} \quad (28)
\]

Similarly for a next high gain group, its user will be selected for the pair according to (28) with respect to user \(j\) and so on up till the highest gain group. Conversely, \(i\) is paired with a user \(k\) from immediate previous low gain group using (29)

\[
\text{Pair}_l = \{i, k | k = \max(G_{ik})\} \quad (29)
\]

For the next low gain group, its user will be selected for the pair according to (29) with respect to user \(k\) and so on up till the lowest gain group.

Consider the simplest case where number of users in each group are same and uniformly distributed. Let \(|h_{\text{ref}, i}|^2\) denote the \(i\)th user of the \(g\)th group. The set of \(M\)-users in a pair is given by (30)

\[
\text{Pair}_l = \{h_{\text{ref}, i}|^2 : \forall \ 1 \leq g \leq G\} \quad (30)
\]

which means that the same indexed (corresponding) users of all the groups are combined in a pair. For \(G\) number of groups, pairing results are summarized in (31)

\[
\text{Pair}_l = \begin{pmatrix} 
|h_{1,1}|^2, |h_{2,1}|^2, \ldots, |h_{G,1}|^2 \\
|h_{1,2}|^2, |h_{2,2}|^2, \ldots, |h_{G,2}|^2 \\
... \\
|h_{1,G}|^2, |h_{2,G}|^2, \ldots, |h_{G,G}|^2 \\
\end{pmatrix} 
\]  

This is a generalized result for the UCGD based \(M\)-users pairing with the assumption of same number of users in all groups. Using the UCGD pairing to make a generalized \(M\)-user pairing model provides a basis for pairing the maximum number of users in a pair by keeping the threshold conditions satisfied. It provides an algorithmic level basis for users pairing rather than random selection of users in a pair. Channel gains of users in groups can be arranged
5. EXACT ERGODIC CAPACITY ANALYSIS

This section provides a detailed mathematical analysis in terms of exact ergodic capacity of two paired users considering both perfect and imperfect SIC scenarios.

5.1. Perfect SIC

Consider a near user UE1 and far user UE2 such that $|h_1|^2 > |h_2|^2$. Suppose they are paired with each other over a common bandwidth $B$. Their power allocation factors are in the order $a_1 < a_2$ and $a_1 + a_2 \leq 1$. Assume that the channel over each link is independent Rayleigh flat fading with channel coefficients $h_1 \sim CN(0, \lambda_1) = d_1^{-\nu}$ and $h_2 \sim CN(0, \lambda_2) = d_2^{-\nu}$ for the links BS$\rightarrow$UE1 and BS$\rightarrow$UE2, respectively, where $d$ represents the distance and $\nu$ is the path loss exponent. The achievable data rate of UE1 in the UE1-UE2 pair is given by

$$C_1 = B \log_2 \left(1 + \rho|h_1|^2a_1\right).$$

The data rate achievable by UE2 can be derived as follows:

$$C_2 = B \log_2(1 + \min(\gamma, \gamma_{1\rightarrow2})) = B \log_2 \left(1 + \min \left(\frac{\rho|h_2|^2a_2}{\rho|h_2|^2a_2 + \rho|h_1|^2a_2}, \frac{\rho|h_1|^2a_1 + 1}{\rho|h_1|^2a_1 + 1}\right)\right)$$

$$= B \log_2 \left(1 + \frac{\min(\rho|h_2|^2a_2, \rho|h_1|^2a_1 + 1)}{\min(\rho|h_2|^2a_2, \rho|h_1|^2a_1 + 1)}\right)$$

$$= B \left\{ \log_2 \left(1 + \min \left(\frac{\rho|h_1|^2}{\rho|h_1|^2a_1}, \frac{\rho|h_2|^2}{\rho|h_2|^2a_1}\right)\right) \right\} - \log_2 \left(1 + \min \left(\frac{\rho|h_1|^2}{\rho|h_1|^2a_1}, \frac{\rho|h_2|^2}{\rho|h_2|^2a_1}\right)\right),$$

where $a_2 = 1 - a_1$ is used. The achievable sum rate $C_{12}$ for the UE1-UE2 pair is calculated by

$$C_{12} = C_1 + C_2.$$  

The probability density functions (PDFs) of $X$, $Y$, and $Z$ are given as follows:

$$f_X(x) = (b + c) \exp(-(b + c)x),$$

$$f_Y(y) = (k + l) \exp(-(k + l)y),$$

$$f_Z(z) = k \exp(-kz).$$

Let $X \triangleq \min(|h_1|^2, |h_2|^2) \rho$, $Y \triangleq \min(|h_1|^2, |h_2|^2) \rho a_1$, and $Z \triangleq \rho|h_1|^2a_1$. The cumulative distribution functions of $X$, $Y$, and $Z$ can be represented as

$$F_X(x) = 1 - \exp \left(-\frac{x}{\lambda_1 \rho}\right) \exp \left(-\frac{x}{\lambda_2 \rho}\right),$$

$$F_Y(y) = 1 - \exp \left(-\frac{y}{\lambda_1 \rho a_1}\right) \exp \left(-\frac{y}{\lambda_2 \rho a_1}\right).$$

Using the PDFs derived for $X$, $Y$, and $Z$, the ergodic capacity for UE1 and UE2 can be calculated as follows. For UE1, the ergodic capacity is given by

$$C_{1\text{erg}}^1 = Bk \int_0^\infty \log_2(1 + z) \exp(-kz)dz$$

$$= -B \frac{\text{Ei}(-(kz)) \exp(k)}{\ln 2},$$

where $\text{Ei}(\cdot)$ represents the exponential integral function. Similarly, the ergodic capacity for UE2 can be written as

$$C_{2\text{erg}}^2 = Bk \int_0^\infty \log_2(1 + x) \exp(-(b + c)x)dx$$

$$= B \left\{ \exp(-(b + c)x) \exp(b + c) \right\}$$

Therefore, the exact ergodic sum capacity for the UE1-UE2 pair can be calculated as

$$C_{12\text{erg}}^\text{exact} = C_{1\text{erg}}^1 + C_{2\text{erg}}^2.$$  

The result in (43) is for the case when perfect SIC is considered at near user receiver.

5.2. Imperfect SIC

Normally, it is assumed that the near user can perfectly decode and cancel the signal of far user. This is based on the assumption of good pairing, so that the differences between channel gains and power allocations of both users are large enough for efficient detection (perfect SIC) of far user’s signal at the near user. But for the bad pairing case, when the difference in channel gains of both users is less, the corresponding closeness in the power allocation of both users causes confusion (interference) for the SIC process at near user. In such scenario, the achievable data rate of UE1 in the UE1-UE2 pair is given by

$$F_z(z) = 1 - \exp \left(-\frac{z}{\lambda_1 \rho a_1}\right).$$

The probability density functions (PDFs) of $X$, $Y$, and $Z$ are given as follows:

$$f_X(x) = (b + c) \exp(-(b + c)x),$$

$$f_Y(y) = (k + l) \exp(-(k + l)y),$$

$$f_Z(z) = k \exp(-kz).$$
\[ \hat{C}_1 = B \log_2 \left( 1 + \frac{|h_1|^2 a_1}{\rho + \Upsilon} \right) \]
\[ = B \log_2 \left( 1 + \frac{\rho|h_1|^2 a_1}{1 + \rho\Upsilon} \right), \tag{44} \]

where \( \Upsilon \) represents the interference due to low-gain user’s signal that may not be canceled perfectly. This interference causes imperfections in the SIC process, which ultimately degrades the achievable capacity. This interference can be defined as a function of the channel gain difference of the paired users or their allocated power factors difference. Let \( \bar{\Upsilon} \triangleq \frac{\rho a_1}{1 + \rho \Upsilon} |h_1|^2 \); the cumulative distribution function of \( \bar{\Upsilon} \) can be written as follows:
\[ F_{\bar{\Upsilon}}(\bar{\Upsilon}) = 1 - \exp \left( -\frac{(1 + \rho\Upsilon)\bar{\Upsilon}}{\lambda_1 \rho a_1} \right). \tag{45} \]

The PDFs of \( \bar{\Upsilon} \) can thus be written as follows:
\[ f_{\bar{\Upsilon}}(\bar{\Upsilon}) = \bar{k} \exp \left( -\bar{k}\bar{\Upsilon} \right), \tag{46} \]
where \( \bar{k} = \frac{1 + \rho\Upsilon}{\lambda_1 \rho a_1} \). The ergodic capacity for UE-1 can thus be calculated as follows:
\[ \hat{C}_{\text{erg}}^1 = B \int_{0}^{\infty} \bar{k} \log_2 \left( 1 + \exp \left( -\bar{k}\bar{\Upsilon} \right) \right) d\bar{\Upsilon} \]
\[ = -B \left( -\frac{\bar{k}}{\ln 2} \right) \exp \left( \frac{\bar{k}}{\ln 2} \right). \tag{47} \]

For UE2, the ergodic capacity calculated for perfect SIC can be used as it is because UE2 does not perform SIC. Therefore, the ergodic sum capacity for the UE1-UE2 pair considering imperfect SIC can be calculated as
\[ \hat{C}_{\text{erg}}^{12} = \hat{C}_{\text{erg}}^1 + C_{\text{erg}}^2. \tag{48} \]

6. SIMULATION RESULTS

Simulations are performed to evaluate the performance of proposed pairing schemes compared with the conventional near–far pairing. System overall capacity comparisons are performed using both perfect and imperfect SIC cases.

In Figure 4, we present the analytical and simulation results for ergodic sum capacity of two users per pair case by considering perfect and imperfect SIC. The distance between BS and UE2 is normalized to unity; that is, \( d_2 = 1 \). For the simplicity of presentation, BS, UE1, and UE2 are assumed to be collinear with \( d_1 = 0.5 \). Moreover, we consider \( v = 4, B = 1 \text{ Hz}, a_1 = 0.1, \) and \( a_2 = 0.9 \). Here, \( \rho \) varies from 0 to 45 dB. For the imperfect SIC case, we assume \( \Upsilon = -25 \text{ dB} \) to show the impact of interference that will cause imperfections in the SIC process. It is evident from Figure 4 that ergodic sum capacity of the two user pair considering perfect SIC shows better performance compared with the imperfect SIC case, particularly at high \( \rho \). It is also noted that, for the imperfect SIC case, the ergodic sum capacity gradually increases up to a certain \( \rho \). After that, the ergodic sum capacity remains almost constant when \( \rho \) exceeds that certain value because of the interference terms in the denominator of (44). A perfect harmony between analytical and the simulation results proves the legitimacy of our analysis.

Based on the analysis in Section 5 and the results shown in Figure 4, we further perform the capacity comparisons for a system of 24 users in a cell where each pair consists of two users. These capacity comparisons are performed between orthogonal frequency-division multiple access (OFDMA), conventional NOMA near–far pairing, and proposed pairing schemes by considering perfect and imperfect SIC as depicted in Figures 5 and 6, respectively. Throughout the simulations, the abbreviations OFDMA and OFDM are used interchangeably. Distribution of users across the cell is uniform. For these two figures, we considered normalized channel gains between 0 and 1. Further, we let \( B = 1 \text{ MHz} \); \( \rho \) varies from 0 to 30 dB, and normalized power per pair is 1; that is, \( a_1 + a_2 = 1 \). Within a pair, fractional transmit power was used to define \( a_1 \) and \( a_2 \). So for two users with gains \( |h_1|^2 \) and \( |h_2|^2 \) in a pair, \( a_1 = \frac{|h_1|^2}{|h_1|^2 + |h_2|^2} \) and \( a_2 = 1 - a_1 \). Thus, low-gain users get high power and vice versa. Furthermore, for imperfect SIC, we consider very small interference as an inverse relation to the channel gain difference between users in a pair.

For the perfect SIC case, it can be seen in Figure 5 that capacities of proposed pairing schemes are only slightly better than the conventional pairing. This is because losses due to SIC imperfections for cell mid pairs in conventional near–far pairing have not been considered. But for the imperfect SIC case in Figure 6, the difference in capacities is evident from the graph. It can be seen that there is very little capacity decrease for UCGD, some decrease for hybrid pairing but great decrease for conventional scheme in Figure 6 compared with Figure 5 even when a small interference is considered.
Figure 5. Cell capacities versus transmit signal to noise ratios (SNRs) (bandwidth = 1 MHz, users = 24, perfect successive interference cancellation). UCGD, uniform channel gain difference; OFDM, orthogonal frequency-division multiplexing.

Figure 6. Cell capacities versus transmit signal to noise ratios (SNRs) (bandwidth = 1 MHz, users in cell = 24, imperfect successive interference cancelation). UCGD, uniform channel gain difference; OFDM, orthogonal frequency-division multiplexing.

The per user capacity comparison of all pairing schemes has been performed in Figure 7 to analyze the interference and SIC effect on individual user capacities. The same parameters of Figures 5 and 6 are used here. Users on the left and right sides on the x-axis of the graph have low and high channel gains, respectively. Users in the center are cell mid users with close channel gains. The capacity variations of users in different regions follow the same reasoning as described earlier throughout the work.

Furthermore, capacity analysis when the number of users in a pair is increased is shown in Figures 8 and 9. The same parameters that were used in Figures 5 and 6 are used here. The total number of users is still 24. The same total power of the system is distributed equally among the pairs. Within a pair with more than two users, similar formulas are used to allocate the power as were used in Figures 5 and 6.

It is clear from Figure 8 that as the number of users in a pair increases, the overall capacity of the system increases correspondingly. For the imperfect SIC case in Figure 9, as the number of per pair users increases, the inter-user channel gain difference becomes less. So, the impact of high interference comes into play, thereby reducing the...
overall capacity gain. If the capacities in Figure 8 are compared with results of Figure 9, it is evident that as the number of users in a pair are increased, the overall capacity gain is reduced for the imperfect SIC case.

Moreover, as the number of users in a pair increases, the end users suffer a lot. For example, in an $M$-user pair, the highest gain user will have to perform SIC for the signals of all $M-1$ in-pair users, which raises questions on the device computational power and the quality of SIC receiver used. Similarly, the lowest gain user will treat the data of all $M-1$ in-pair high-gain users as noise. This will severely affect the data rates of low-gain users. These aspects point towards the need for an upper bound on the number of users in a pair.

7. CONCLUSION

In this paper, effects of near–far user pairing on the performance of cell center, mid, and edge users have been investigated. It has been pointed out that as the near and far users are paired in conventional near–far pairing, the cell mid users are left unpaired. If these users are paired with each other, the small difference in their channel gains and allocated powers causes imperfections in the SIC, which degrades their capacity ultimately. On the other hand, if these users are left unpaired to be served with MA, the SIC issue can be avoided, but these users cannot benefit from the capacity gains provided by NOMA.

Therefore, two users pairing strategies are proposed that can accommodate all the users in pairs in an intelligent manner, so that the SIC imperfection issue can also be avoided or minimized. It has been shown that the proposed schemes give better capacity results compared with conventional near–far pairing, especially when imperfect SIC is considered. It is also shown that as the number of in-pair users increases, the SIC imperfection effects increase because of decreasing channel gain difference of in-pair users. Furthermore, for a large number of users in a pair, the highest gain users will have to decode and cancel the signals of so many users through SIC, which will be challenging for the devices. Similarly, the lowest gain user will face large amount of noise. This places upper bound on the maximum number of users in a pair.

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**AUTHORS’ BIOGRAPHIES**

Muhammad Basit Shahab received his BS in Electrical Engineering from University of Engineering and Technology (UET) Lahore, Pakistan. His MS in Electrical Engineering was from University of Management and Technology (UMT) Lahore, Pakistan. Currently, he is working as graduate research assistant at Wireless and Emerging Network System Lab, Kumoh National Institute of Technology, South Korea. His main research areas are radio access technologies (RATs) for wireless communications, non orthogonal multiple access (NOMA), ultra-dense cells, LTE/LTE-A, and smart grids.

Mohammad Irfan received his BSc degree in Electrical and Electronic Engineering from Islamic University of Technology, Dhaka, Bangladesh, in 2011. Currently, he is working as graduate research assistant at wireless and Embedded Networking System Lab, while attending graduate school at Kumoh National Institute of Technology, South Korea. His main research area includes Orthogonal/Non-orthogonal multiple access, MIMO, and new information carrying domains for wireless communications.

Md Fazul Kader was born in 1982. He received his BSc and MSc in Computer Science and Engineering (CSE) from the Chittagong University of Engineering and Technology (CUET), Bangladesh, in 2005 and 2014, respectively. Currently, he is working towards a PhD at the WENS Lab., Kumoh National Institute of Technology, South Korea. From 2007 onwards, he is a faculty member of the Department of Applied Physics, Electronics, and Communication Engineering, University of Chittagong, Bangladesh. His major research interests include cognitive radio networks, cooperative communications, MIMO, computer networks, NOMA and spatial modulation.

Soo Young Shin received his BS, MS, and PhD degrees in Electrical Engineering and Computer Science from Seoul National University, Korea in 1999, 2001, and 2006, respectively. He was a visiting scholar in FUNLab at University of Washington, US, from July 2006 to June 2007. After 3 years working in WiMAX design lab of Samsung Electronics, he is now assistant professor in School of Electronics in Kumoh National Institute of Technology since September 2010. His research interests include wireless LAN, WPAN, WBAN, wireless mesh network, sensor networks, coexistence among wireless networks, industrial and military network, cognitive radio networks, and next generation mobile wireless broadband networks.