7 Speckle Metrology

In the preceding chapters the speckles which always appear when coherent light is diffusely scattered or reflected were treated as a disturbance to be suppressed or eliminated. On the other hand in coherent optical metrology the speckles can be viewed as the fundamental carriers of information, and thus can be used for specific measurement techniques. A number of methods of speckle metrology are closely related to methods of holographic interferometry. Therefore holographic interferometry and speckle metrology are often presented together in a closed form. A typical example for the close relation between holographic and speckle interferometry is the ESPI/DSPI method, which by several authors is regarded as image plane holography and therefore a holographic interferometric method. For these reasons in the following a brief introduction to the main techniques of speckle metrology is given.

Two principal approaches have to be distinguished in speckle metrology applied to e. g. deformation analysis of opaque diffusely reflecting surfaces: In speckle photography two reflected speckle fields are incoherently superposed to give information about an in-plane displacement; in speckle interferometry two interference fields are compared, each one generated by coherent superposition of the reflected wave field and a reference wave. The two fields to be compared correspond to the object states before and after the deformation.

The nature of the speckles and their statistics, which are of general interest also for holographic interferometry are described in detail in Section 2.5.

7.1 Speckle Photography

In speckle photography an opaque diffusely reflecting surface is illuminated by coherent light. The resulting speckle pattern is imaged by the lens of a photo-camera onto photographic film. The exposure results in a pointwise blackening of the film. When the surface point motion has a component in a direction normal to the optical axis, the speckle pattern follows this displacement component. A developed double exposure negative with the two exposures before and after the deformation will consist of a manifold of speckle pairs. The distance between the points of each pair is proportional to the displacement component normal to the optical axis of the corresponding object point, the direction of this lateral displacement component is the same as the direction of the shift of the related speckles.

For reconstruction the double exposure negative, often called specklegram, is illuminated in a pointwise manner by an unexpanded laser beam, Fig. 7.1. The point pairs in the small region where the beam passes the specklegram act like the two apertures in Young’s double aperture interferometer, see Section 2.3.2. We get parallel equispaced fringes with a spacing of \( \frac{\lambda L}{d} \) at a screen, which is placed at a distance \( L \) from the specklegram. \( d \) is the distance...
in the pointpair in the specklegram, \( \lambda \) the wavelength used. The orientation of the fringes is orthogonal to the direction of the measured displacement component. By scanning the laser spot over the specklegram a two-dimensional field of displacement components can be measured. It should be stressed that from a single specklegram one can get only two pieces of information at each point: The modulus of the displacement projection onto a plane orthogonal to the optical axis, and the direction of this displacement component. The first is given by the distance of the Young’s fringes, the second by the orientation of the fringes. In other words, we obtain the \( x \)- and \( y \)-components of the displacement vector in a coordinate system with the z-axis as the optical axis. There are a number of approaches to determine exactly the spacing and orientation of the fringes [757–759]. Good results have been obtained by locating the primary side lobes in the numerical 2D Fourier spectrum of the fringe pattern [760, 761].

Contrary to the pointwise evaluation and scanning there is a full field measurement method [164], where the specklegram is placed in an optical Fourier processor. A screen with a small hole is placed in the diffraction plane and passes only a small part of the spectrum, ideally a single spatial frequency. The resulting pattern in the image plane after such a filtering shows contours of equal displacement components in the direction given by the position of the filter. The filter position in the spatial frequency domain can now be varied by shifting the hole. It is obvious that in this way the measuring sensitivity can be varied, even after the speckle patterns have been originally recorded.

The main sensitivity of speckle photography is in the direction normal to the optical axis, that means for in-plane displacements of the surface points. The displacements generally have to be larger than the speckle size, which can be controlled by the aperture of the photo-camera.

### 7.2 Electronic and Digital Speckle Interferometry

The most important technique of speckle interferometry is the digital speckle pattern interferometry (DSPI), originally called electronic speckle pattern interferometry (ESPI), sometimes named electronic holography [762], TV holography [763–765], or electro-optic holography [516, 766, 767], or even digital holography without wavefront reconstruction [225] as well as digital image plane holography [768–771]. It was invented independently by several
groups [46–48]. The original aim was to overcome the time consuming wet-chemical processing of the silver halide holograms and to use electronic camera tubes instead. To adapt the micro-interference between object and reference wave to the resolution of the cameras, colinear reference and object waves have to be employed and an imaging system has to be used, Fig. 7.2. The object surface is focused onto the camera target, which in conjunction

![Figure 7.2: Arrangement for digital speckle interferometry.](image)

with the colinear reference wave results in large speckles which now can be resolved by the camera but degrade the resulting interference pattern. This disadvantage is accepted due to the nearly real-time recording and reconstruction, and becomes less severe as CCD-targets get more and smaller pixels. Due to the focusing of the object’s surface onto the camera target we record an image plane hologram, contrary to digital holography where only the Fresnel or Fraunhofer diffraction field of the object wavefront is registered, as defined in Chapter 3.

The object wave field in the image plane \((x, y)\), the plane of the camera target, Fig. 7.2, can be described by

\[
E^{(ob)}(x, y) = E_0^{(ob)}(x, y) e^{i\phi^{(ob)}(x, y)} \tag{7.1}
\]

where \(E_0^{(ob)}(x, y)\) is the real amplitude and \(\phi^{(ob)}(x, y)\) is the random phase due to the surface roughness, Fig. 7.3a. The colinear reference wave field

\[
E^{(ref)}(x, y) = E_0^{(ref)}(x, y) e^{i\phi^{(ref)}(x, y)} \tag{7.2}
\]

is superposed. This reference wave may be a plane wave, a spherical wave, or an arbitrary reflected one, Fig. 7.3b. Only intensities are recorded by the TV target, Fig. 7.3d

\[
I_A(x, y) = \left| E^{(ob)}(x, y) + E^{(ref)}(x, y) \right|^2 \tag{7.3}
\]

\[
= I^{(ob)}(x, y) + I^{(ref)}(x, y) + 2 \sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y)} \cos \psi(x, y). \]

This is the recorded, digitized, and stored speckle pattern with the stochastic phase difference \(\psi(x, y) = \phi^{(ob)}(x, y) - \phi^{(ref)}(x, y)\), Fig. 7.3c. A deformation changes the phase \(\phi^{(ob)}(x, y)\) of each point by \(\Delta\phi(x, y)\), Fig. 7.3e, so that the wave field after deformation is

\[
E^{(ob)'}(x, y) = E_0^{(ob)}(x, y) e^{i[\phi^{(ob)}(x, y) + \Delta\phi(x, y)]}. \tag{7.4}
\]
Superposition with the colinear reference wave $E^{(\text{ref})}(x, y)$ leads to $I_B(x, y)$, Fig. 7.3f

$$I_B(x, y) = I^{(\text{ob})}(x, y) + I^{(\text{ref})}(x, y) + 2\sqrt{I^{(\text{ob})}(x, y) I^{(\text{ref})}(x, y) \cos[\psi(x, y) + \Delta\phi(x, y)]}. \quad (7.5)$$

In the digital image processing system this second speckle pattern $I_B(x, y)$ is subtracted in a pointwise manner in real time from the stored $I_A(x, y)$, where it is assumed that the deformation changes the phase but not the amplitude, meaning $I^{(\text{ob})}(x, y) = I^{(\text{ob})}(x, y)$. The resulting difference is, Fig. 7.3g

$$(I_A - I_B)(x, y) = 2\sqrt{I^{(\text{ob})}(x, y) I^{(\text{ref})}(x, y) \cos \psi - \cos \psi \cos \Delta\phi + \sin \psi \sin \Delta\phi}[x, y] y \sin \Delta\phi(x, y) y. \quad (7.6)$$

To display this result in real time on a monitor, positive intensities are obtained by taking the modulus $|I_A - I_B|$ or the square $(I_A - I_B)^2$, Fig. 7.3h. The square root in (7.6) describes the background illumination. The first sine-term gives the stochastic speckle noise which varies randomly from pixel to pixel. This noise is modulated by the sine of the half phase difference induced by the deformation. This low frequency modulation of the high frequency speckle noise is recognized as an interference pattern. The relation between the displacement vector $d(x, y)$ and the phase difference $\Delta\phi(x, y)$ is as in holographic interferometry, see (4.20) and (4.21)

$$\Delta\phi(x, y) = \frac{2\pi}{\lambda} d(x, y) \cdot [b(x, y) - s(x, y)]. \quad (7.7)$$

An alternative way for determining the phase difference from two speckle interferograms is the Fourier transform method first suggested in [457]. Here both intensities $I_A(x, y)$ and
$I_B(x, y)$ are Fourier transformed. The amplitude spectra in the first and minus first diffraction orders – the first two side-lobes – resemble the aperture, see Fig. 7.2. If the lateral placement of the aperture and the position of the virtual reference source point are properly chosen, the side-lobes do not overlap [306] and a bandpass filter as described in Section 5.6.1 can be applied. A virtual reference source point outside the aperture generates carrier fringes as described in Section 5.6.5. After one side-lobe is isolated by bandpass filtering an inverse Fourier transform is applied and gives complex $c_A(x, y)$ and $c_B(x, y)$. From these the interference phase $\Delta \phi(x, y)$ is calculated as defined in (5.71) by

$$
\Delta \phi(n, m) = \arctan \frac{\text{Re}\{c_A(x, y)\}\text{Im}\{c_B(x, y)\} - \text{Re}\{c_B(x, y)\}\text{Im}\{c_A(x, y)\}}{\text{Re}\{c_A(x, y)\}\text{Re}\{c_B(x, y)\} + \text{Im}\{c_A(x, y)\}\text{Im}\{c_B(x, y)\}}
$$

$$
= \arctan \frac{\text{Re}\{c_B(x, y)c_A^*(x, y)\}}{\text{Re}\{c_B(x, y)c_A^*(x, y)\}}.
$$

(7.8)

This approach elucidates the close relationship to digital holography, therefore some authors use the term digital image plane holography [768–770].

Equation (7.7) shows that maximum sensitivity of DSPI/ESPI is for out-of-plane displacements. For measuring transversal displacements a modified setup using two illuminations from opposite directions having equal angles to the surface normal is recommended [582, 697, 772]. DSPI/ESPI patterns essentially contain the same information as the corresponding holographic interferograms. Thus their production requires the same precautions concerning vibration isolation and stability during the recording process. The results can be observed in real time, due to the electronic recording there is no problem with an exact repositioning of a hologram plate. The interference phase map quality can be improved by filtering in the spatial or spatial frequency domain [773].

While most applications of digital speckle interferometry are in deformation measurements of opaque surfaces these methods can also be successfully applied in fluid mechanics. Now the flowing transparent medium is seeded with tracer particles which constitute the reflecting object [267]. The particle field may be illuminated from any convenient direction; however, a useful approach is illumination by a thin light sheet as in particle image velocimetry (PIV).

The measurement of transient phenomena like impact studies, vibration analysis, or flow diagnostics is possible using double-pulsed illumination and recording. In this way mechanical amplitude and phase of surface acoustic waves have been measured [774] with interference phase determination by the Fourier transform method. While in this application the surface acoustic waves constitute a spatial carrier frequency by themselves, carrier fringes can be introduced by a tilt between object and reference wave. This allows the use of Fourier transform evaluation, e.g. in the double pulse measurement of brake squeal [775]. The separation of vibration modes using four pulses of a Q-switched ruby laser is described in [776, 777]. Here the method is called digital holography but since the “laser light scattered by the object is collected with an imaging lens that forms the image of the object on each of the three CCD camera faceplates” [776] there clearly an electronic speckle pattern interferometer with out-of-plane sensitivity is employed.
7.3 Electro-optic Holography

Electro-optic holography, also known as electronic holography or TV holography, is a combination of phase stepping and digital speckle interferometry [49, 50, 202]. For static measurements n phase stepped speckle patterns are recorded in the unstressed and n phase stepped speckle patterns in the stressed state. The recorded intensities are, see (7.3)

\[
I_n(x, y) = I^{(ob)}(x, y) + I^{(ref)}(x, y) + 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \cos[\psi(x, y) + \phi_{Rn}]} \\
I'_n(x, y) = I^{(ob)'}(x, y) + I^{(ref)}(x, y) + 2\sqrt{I^{(ob)'}(x, y) I^{(ref)}(x, y) \cos[\psi(x, y) + \Delta \phi(x, y) + \phi_{Rn}]}.
\]

The notation is as in Section 7.2, \(\phi_{Rn}\) are the phase shifts. While generally arbitrary phase shifts \(\phi_{Rn}\) can be employed, see Section 5.5, the most used are \(\phi_{R1} = 0^\circ\), \(\phi_{R2} = 90^\circ\), \(\phi_{R3} = 180^\circ\), and \(\phi_{R4} = 270^\circ\). This results in

\[
\begin{align*}
I_1(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) + 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \cos[\psi(x, y)]} \\
I_2(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) + 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \sin[\psi(x, y)]} \\
I_3(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) - 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \cos[\psi(x, y)]} \\
I_4(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) - 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \sin[\psi(x, y)]}.
\end{align*}
\]

and

\[
\begin{align*}
I'_1(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) + 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \cos[\psi(x, y) + \Delta \phi(x, y)]} \\
I'_2(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) + 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \sin[\psi(x, y) + \Delta \phi(x, y)]} \\
I'_3(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) - 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \cos[\psi(x, y) + \Delta \phi(x, y)]} \\
I'_4(x, y) &= I^{(ob)}(x, y) + I^{(ref)}(x, y) - 2\sqrt{I^{(ob)}(x, y) I^{(ref)}(x, y) \sin[\psi(x, y) + \Delta \phi(x, y)]}.
\end{align*}
\]

These systems of equations can be solved, see (5.24), yielding \(\psi(x, y)\) and \(\psi(x, y) + \Delta \phi(x, y)\), whose difference is the interference phase distribution \(\Delta \phi(x, y)\). The advantage over conventional DSPI-patterns becomes obvious, when

\[
\sqrt{[(I_1 - I_3) + (I'_1 - I'_3)]^2 + [(I_2 - I_4) + (I'_2 - I'_4)]^2} = 8\sqrt{I^{(ob)} I^{(ref)} \cos(\Delta \phi/2)}
\]

is displayed. This interference pattern exhibits much less speckle noise than DSPI-patterns.

Besides static deformation measurement electro-optic holography also has been effectively applied to sinusoidally vibrating objects [376]. High quality time average interference patterns can be synthesized from phase stepped recordings. The argument of the Bessel function is determined with high accuracy if the phase of the object or reference wave is modulated at the same frequency and phase as the object vibration [376, 762].

In the described method the four phase shifted patterns were taken one after the other. To get several phase shifted patterns not sequentially in time but spatially separated, the in-line reference wave of digital speckle interferometry must be tilted, which is accomplished by
shifting the focus of the reference wave slightly out of the focus of the camera lens. In [778] this shift was adjusted to produce phase shifts of 120° between subsequent pixels. The three phase shifted images now are coded in neighboring pixels. If the speckles are large enough – about three pixels per speckle – and if the phase to be measured does not vary too much over the speckle size, then the interference phase can be determined for each triple of neighboring pixels by
\[
\Delta \phi = \arctan \left( \sqrt{3} \frac{I_3 - I_2}{2I_1 - I_2 - I_3} \right),
\]
see Table 5.1. If the angle between object wave and the reference wave tilted in the x-direction is \( \theta \), the image plane hologram can be interpreted as a spatial carrier modulated by the object information [779]. The spatial carrier frequency is
\[
f_0 = \frac{\sin \theta}{\lambda}.
\]
Let the pixel pitch be \( \Delta x \), then provided the interference phase \( \Delta \phi \) changes slowly, we can calculate \( \Delta \phi \) in a pointwise manner by
\[
\Delta \phi(x, y) = \arctan \left( \frac{I(x - \Delta x, y) - I(x + \Delta x, y)}{I(x - \Delta x, y) + I(x + \Delta x, y) - 2I(x, y) \tan(\pi \Delta x f_0)} \right).
\]
(7.12)

A comparison of this approach with digital holography using the Fresnel diffraction field instead of the image plane hologram is given in [64].

The simultaneous use of several reference waves tilted in different directions with respect to the object wave defines various spatial carriers. Each reference wave can be written as [62]
\[
R_k(\xi, \eta) = r_k(\xi, \eta) \exp\{-2\pi i(f_k\xi + f_k\eta)\}. \tag{7.13}
\]

Here the \((f_k, f_k)\) describe the spatial carrier frequencies. By considering the dimensions of the imaging aperture and the location of the reference point sources relative to this aperture, the carrier frequencies are separated from each other in the spatial frequency domain and can be isolated by proper filtering. This allows the application of Fourier transform evaluation with carrier [43] as introduced in Section 5.6.1. In this way the three deformation components of a vibrating object are discriminated and measured with simultaneous recording of digital image plane holograms [62].

Furlong [239] uses this technique, there called optoelectronic holography (OEH), in conjunction with an optoelectronic holographic microscope (OEHM) to characterize shape and deformation of microelectromechanical systems (MEMS).

### 7.4 Speckle Shearography

The requirement of vibration isolation can be dropped to a good extent when using the speckle-shearing methods. Here only the spatial variations of the displacement in a predetermined direction are measured, so the methods are rather insensitive to rigid body motions [136, 780]. In the following the method is explained with the example of digital shearography, which has attained some economic importance.

Again two wave fields, object and reference, as in DSPI interfere, but now two slightly spatially shifted speckle fields of the rough surface are superposed. The role of the reference wave is taken by one of the two mutually shifted object wave fields, one speaks of self-reference. The shifting of the speckle fields is performed by a shearing element, e. g. a glass wedge in front of one half of the imaging lens, two tilted glass plates, a Wollaston prism, or a Michelson interferometer-like arrangement with one mirror slightly tilted, Fig. 7.4.
Let the two wave fields be separated laterally by the mutual shearing $\Delta x$, then we get the two nearly colinear wave fields

$$
E_1(x, y) = E_{01}(x, y) e^{i\phi(x, y)}
$$

$$
E_2(x, y) = E_{02}(x, y) e^{i\phi(x + \Delta x, y)}.
$$

(7.14)

Their interference produces the speckle pattern

$$
I_A(x, y) = |E_1(x, y) + E_2(x, y)|^2
$$

$$
= I_1(x, y) + I_2(x, y) + 2\sqrt{I_1(x, y) I_2(x, y)} \cos \psi(x, y)
$$

(7.15)

which is recorded, digitized, and stored. $\psi(x, y)$ is the randomly distributed phase difference $\psi(x, y) = \phi(x, y) - \phi(x + \Delta x, y)$. Deformation of the object leads to the wave fields

$$
E_3(x, y) = E_{01}(x, y) e^{i[\phi(x, y) + \Delta \phi(x, y)]}
$$

$$
E_4(x, y) = E_{02}(x, y) e^{i[\phi(x + \Delta x, y) + \Delta \phi(x + \Delta x, y)]}
$$

(7.16)

whose superposition yields the speckle pattern

$$
I_B(x, y) = I_1(x, y) + I_2(x, y)
$$

$$
+ 2\sqrt{I_1(x, y) I_2(x, y)} \cos[\psi(x, y) + \Delta \phi(x, y) - \Delta \phi(x + \Delta x, y)].
$$

(7.17)
Pointwise subtraction gives
\[
(I_A - I_B)(x, y) = 2\sqrt{T_1 T_2} \left\{ \cos[\psi(x, y)] - \cos[\psi(x, y) + \Delta \phi(x, y) - \Delta \phi(x + \Delta x, y)] \right\}
\]
\[
= 4\sqrt{T_1 T_2} \sin \left[ \frac{\psi(x, y)}{2} + \frac{\Delta \phi(x, y) - \Delta \phi(x + \Delta x, y)}{2} \right] \cdot \sin \frac{\Delta \phi(x, y) - \Delta \phi(x + \Delta x, y)}{2}.
\]
(7.18)

Positive values may be obtained by taking the modulus or square. Again the term in the root is the background, the first sine-term is the stochastic speckle noise modulated by the second sine-term which stems from the deformation. The displacement vector field \(d(x, y)\) is contained in the argument of this sine by
\[
\frac{\Delta \phi(x, y) - \Delta \phi(x + \Delta x, y)}{2}
\]
\[
= \frac{\pi}{\lambda} \left\{ d(x, y) \cdot [b(x, y) - s(x, y)] - d(x + \Delta x, y) \cdot [b(x + \Delta x, y) - s(x + \Delta x, y)] \right\}
\]
\[
\approx \frac{\pi}{\lambda} [d(x, y) - d(x + \Delta x, y)] \cdot [b(x, y) - s(x, y)]
\]
\[
= \frac{\pi}{\lambda} \left[ \frac{d(x, y) - d(x + \Delta x, y)}{\Delta x} \right] \cdot [b(x, y) - s(x, y)] \Delta x
\]
\[
\approx \frac{\partial d(x, y)}{\partial x} \frac{\pi \Delta x}{\lambda} [b(x, y) - s(x, y)].
\]
(7.19)

What produces the interference pattern is an approximation to the derivative of the displacement field in the direction of the image shearing, here the \(x\)-direction. For rigid body motions we have \(d(x, y) = \text{const}\), implying \(\partial d(x, y)/\partial x = 0\). This explains the insensitivity of the shearing methods to rigid body motions. The sensitivity of the method can be adjusted by controlling the amount of shearing. Besides the shearing in the \(x\)-direction we can shear in the \(y\)- or other angular directions. Also radial shearing, rotational shearing, inversion shear, or reversal shear can be performed with related optical arrangements [781].

Figure 7.5a presents a DSPI-image of a plate subjected to an out-of-plane deformation by pressing at a central point from the plate’s back side. Thus a deformation in the form of a Gaussian bell is produced. The fringes can be interpreted as equi-height lines of the deformed object. For comparison in Fig. 7.5b the same object undergoing the same deformation is investigated by digital shearography. The shear is in the horizontal direction, so the fringes can be explained as equi-contours of the derivative in the horizontal direction with highest fringe order at the locus of steepest slope of the hill. At the top of the hill we have zero slope and thus zero fringe order in the shearographic pattern.

A one-dimensional simulation of speckle shearography is shown in Fig. 7.6. Figure 7.6a displays the random phase \(\phi(x)\), while Fig. 7.6b gives the intensity \(I_A(x)\) resulting from interference between sheared and unsheared wave fields focused onto the recording target in the image plane. Figure 7.6c shows the assumed additional phase \(\Delta \phi(x)\) due to the deformation and in Fig. 7.6d the recorded intensity \(I_B(x)\) after deformation is seen. The difference \(I_A(x) - I_B(x)\) of the two recorded intensities is given in Fig. 7.6e, and its modulus \(|I_A(x) - I_B(x)|\) in Fig. 7.6f.
Equations (7.18) and (7.19) show that the fringe density in shearographic patterns depends on the wavelength $\lambda$ of the light used. So multiple wavelengths, especially when using broadband sources, would decrease the fringe modulation until their total deterioration. However, Falldorf [782] has indicated how the shift of the fringes due to $\lambda$ can be compensated by employing a dispersive optical component. This component used in [782] is a wedge prism, arranged under a certain angle to the optical axis, that produces a shear $\Delta x$ proportional to $\lambda$. So the factor $\Delta x/\lambda$ in (7.19) becomes a constant, and fringe densities in shearographic measurements are independent from the wavelengths used, provided that the deformation along the shear is linear to the first order.