Adaptive neural network control of bilateral teleoperation with unsymmetrical stochastic delays and unmodeled dynamics

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SUMMARY

In this paper, adaptive NN control is proposed for bilateral teleoperation system with dynamic uncertainties, unknown external disturbances, and unsymmetrical stochastic delays in communication channel to achieve transparency and robust stability. Compared with previous passivity-based teleoperation framework, the communication delays are unsymmetrical and stochastic. By partial feedback linearization using nominal dynamics, the nonlinear dynamics of the teleoperation system are transformed into two subsystems: local master/slave dynamics control and time-delay motion tracking. By integrating Markov jump systems and adaptive parameters updating, adaptive NN control strategy is developed. The stability of the closed-loop system and the boundedness of tracking errors are proved using Lyapunov–Krasovskii functional synthesis under specific linear matrix inequalities conditions. The proposed adaptive NN control is robust against motion disturbances, parametric uncertainties, and unsymmetrical stochastic delay, which effectiveness is validated by extensive simulation studies. Copyright © 2013 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Bilateral teleoperators are defined as electromechanical mechanism that enable humans to move, sense, and physically manipulate objects at a distance by the exchange of position, velocity, and/or force information [1]. Because of the difficulty posed by the presence of a communication channel with random transmission delays [2], bilateral teleoperation is one of the most interesting and challenging research areas, which can be used in a much wider range of applications such as outer space exploration [3], handling of toxic materials [4], and minimally invasive surgery [5]. Bilateral teleoperation not only transmits the master motion and/or force to the slave site but also receives the feedback of the motion and/or force information from the slaver. With the motion of the slave and the perception of force feedback, the user can properly interact with the remote environment and perform many operation tasks.

In bilateral teleoperation, the master and the slave robots are connected via a communication channel, which often involves large distances or imposes limited data transfer between the local and the remote sites. Therefore, time delay may happen between the time when a command is generated by the operator and the time when the command is executed by the remote robots. It is
well known that time delays in the communication channel can destabilize the whole teleoperation system if they are not well compensated for [6, 7]. It is even worse for the networked teleoperation, for example, internet-based teleoperation, in this case, the delays are stochastic, irregular, and unsymmetric [8, 9].

Many control methods in the past years have been proposed to deal with the time-delay in network-based teleoperation, especially, the approaches based on the passivity theory are prevalent and have been extensively studied [10–12]. In [13], passivity-based teleoperation was proposed where passivity and scattering theory was used to analyze mechanisms responsible for loss of stability and derive a time delay compensation scheme to guarantee stability independent of the (constant) delay. In [10], the passivity-based architecture was extended to guarantee state synchronization of master/slave robots in free motion. The passivity-based schemes require that both the human operator and the environment are passive. On the other hand, supervisor control was proposed where the human operator supervises the task generating high-level commands [14] and [15]. Such commands are sent to plan and control algorithms implemented on the remote robot, and it is apparent that this method does not assure a continuous teleoperation.

However, in reality, the time delay in communication channel is often stochastic. In [16] and [8], the delay of internet is characterized as random, unbounded, and different for both communication channel in the control loop rather than constant time delay or fixed known time delay [17]. Moreover, in [16], the internet delay can be modeled as stochastic delay with probability distributions governed by an underlying Markov chain, which is also called Markov jumping parameters. Because of the complexity of the communication network, the delays of data packets are also unsymmetrical. The data packets in the forward and backward channels may go through different network paths, leading to the fact that the forward and backward delays are not equal [18]. Therefore, it is very important to investigate the asymmetry of stochastic time delays and their impact on the stability of network-based teleoperation systems.

Recently, many methods based on Markov jump linear systems consider networks with packet loss and varying bounded delays for networked control systems [6, 9, 19–25]. However, two or more separate plants in teleoperation systems must be coordinated, the proposed control in networked control systems cannot be applied directly for teleoperation setting [16]. To the best of our knowledge, there is few work about nonlinear bilateral teleoperation systems with Markov jumping parameters.

Another important factor of bilateral teleoperation is the dynamics uncertainty. Although in [26] and [27], predictive methods can be used where the remote robot is displayed to the human operator, who generates commands interacting with the graphic environment. The precise knowledge of the environment, the operator, or both are required to achieve a good performance. Actually, the precise knowledge of robots, the environment, and the operator are difficult to acquire; therefore, we need to develop an adaptive technique to deal with this problem. NN systems have been considered as powerful tools in robotics controls and applications [28–31], which are capable of dealing with ill-defined dynamics systems and unstructured uncertainties. Nevertheless, it is easy to see that the aforementioned works did not investigate nonlinear teleoperators with dynamics uncertainties and time-varying time delays.

In this paper, we consider adaptive NN control of bilateral teleoperation with stochastic asymmetry time delays. Through the impedance control, the configuration variables of the master and the slave robot are synchronized such that the force reflection can be achieved. In order to achieve robust stability in bilateral teleoperation with stochastic asymmetry time delays, adaptive NN control is investigated for bilateral teleoperation system in the presence of stochastic asymmetry time delays in communication channel, dynamic uncertainty, and unknown external disturbance. By partial feedback, linearization using nominal dynamics, the nonlinear dynamics of the teleoperation system is transformed into two subsystems: local master/slave dynamics control and time delay motion tracking. We propose new adaptive NN control strategies with online parameters adaption to eliminate the nonlinearities and dynamic uncertainties of the master and slave robots. Moreover, LMI based on Markov jump systems is used to stabilize the resulting linearized dynamics. The stability of the closed-loop system and the boundedness of tracking errors are proved using Lyapunov stability synthesis under specific LMI conditions.
2. DYNAMICS DESCRIPTION

Let us consider a teleoperator consisting of a pair of n-DOF nonlinear robotic systems

\[
M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + G_m(q_m) + f_m(\dot{q}_m) = F_h + \tau_m
\]

\[
M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + G_s(q_s) + f_s(\dot{q}_s) = \tau_s + F_e
\]

where \( q_j, \dot{q}_j, j = m, s \) are the \( n \)-dimension vectors of joint position and velocity for a teleoperated robots, \( \tau_m \) and \( \tau_s \) are the applied torques, \( M_j \) is the \( n \times n \) symmetric positive definite robotic inertia matrix, \( C_j(q_j, \dot{q}_j) \) is the \( n \times n \) matrix of Centripetal and Coriolis torque vector, \( G_j(q_j) \) are the gradients of the gravitational potential energy, \( f_j(q_j) \) is external friction vector. Moreover, \( F_h \) and \( F_e \) are human-operator force (torque) and environmental force (torque), respectively.

For the teleoperation system (1) and (2) illustrated in Figure 1, we consider the case that the human operator torque \( F_h \) and the environmental torque \( F_e \) are given by

\[
F_h = -K_{h0} + K_h q_m + C_h \dot{q}_m
\]

\[
F_e = -K_{e0} + K_e q_s + C_e \dot{q}_s
\]

where \( K_{h0}, K_{e0}, K_h, C_h, K_e, \) and \( C_e \) are positive scalars.

Remark 2.1

We can see that the human operator force \( F_h \) contains the constant \( K_{h0} \). For the passive-based approach, the human operator and the environment are passive, that is, \( \int_0^t \dot{q}_m^T F_h dt > 0 \), \( \int_0^t \dot{q}_s^T F_e dt > 0 \) [11]. It is obvious that (3) and (4) cannot satisfy \( \int_0^t \dot{q}_m^T F_h dt > 0 \), \( \int_0^t \dot{q}_s^T F_e dt > 0 \) because \( K_{h0} \) and \( K_{e0} \) exist. Therefore, the passive-based approach cannot be applicable in the teleoperation control, while this paper aims to develop the stability conditions of the system (1) and (2) under nonlinear adaptive control and stochastic time delay.

The control objective can be described as if \( F_h = F_e \), with \( \ddot{q}_m, \ddot{q}_s, \dot{q}_m, \) and \( \dot{q}_s \) converge to zero under the time delay \( d_m(t) \) and \( d_s(t) \), \( d_m(t) \) stands for the forward time delay, whereas \( d_s(t) \) represents the backward time delay. Consider asymmetric time delay for the master-to-slave and slave-to-master communication, then define the coordination errors between the master and slave robots

\[
e_m(t) = e_{m0} + K_h q_m(t) - K_e q_s(t - d_s(t))
\]

\[
e_s(t) = e_{s0} - K_h q_m(t) - K_e q_s(t - d_m(t))
\]

Figure 1. The teleoperation system.
where $K_0 = -K_{h0} + K_{e0}$. Therefore, we have

$$\lim_{t \to \infty} \|e_m\| = 0, \quad \lim_{t \to \infty} \|e_s\| = 0 \quad (7)$$

The delays $d_m(t)$ and $d_s(t)$ through the communication channel are assumed to be mode-dependent time-varying and governed by two Markov processes $\eta_m(t)$ and $\eta_s(t)$, respectively, and $\eta_m(t)$ and $\eta_s(t)$ are two independent continuous-time discrete-state Markov process taking values in a finite set $S = \{1, 2, \ldots, N\}$ with a transition probability matrix given by [16]

$$P(\eta_j(t+\Delta) = l \mid \eta_j(t) = i) = \begin{cases} \pi_{il} \Delta + o(\Delta) & i \neq l \\ 1 + \pi_{ii} \Delta + o(\Delta) & i = l \end{cases} \quad (8)$$

where $j = m, s$, $\Delta > 0$, $\lim_{\Delta \to 0} o(\Delta)/\Delta = 0$, and $\pi_{ii} \geq 0$ for all $i \in S$, $i \neq l$ denotes the transition rate from mode $i$ to $l$. $\pi_{ii} = - \sum_{l=1,l \neq i}^N \pi_{il}$, for all $i \in S$.

**Remark 2.2**

It is quite general that modeling the network delay as a continuous-time Markov process. On the one hand, in real communication systems, current time-delays are usually correlated with the previous time-delays [32]. So, it is reasonable to model random delays as homogeneous Markov process. On the other hand, a real network inevitably exists the network queues and varying network loads; to model those phenomena, the network model needs to have a memory or a state. One way is by letting the distribution of the network delays be governed by the state of an underlying Markov process [33]. That is to say, the model of network delays that use Markov process can reflect the actual network. It can also be noticed that, the network delays that modeled as a general continuous-time time-varying delays can be seen as a special Markov model with only one state.

**Assumption 2.1**

The time-varying delays of the system are time-differentiable for all time and satisfy $d_m(t) \leq d_m (t) \leq d_m, d_s(t) \leq d_s, d_m(t) \leq \mu_m < 1, d_s(t) \leq \mu_s < 1$, where $d_m, d_m, d_s, d_s, \mu_1, \mu_2$ are positive scalars. Furthermore, we define $d = \max[d_m, d_s]$, and $\mu = \max[\mu_m, \mu_s]$, which will be used in the proof of stabilities.

**Remark 2.3 ([34])**

This assumption is less conservative for the time delay in the communication channel. First, the time delays of practical systems are often bounded; thus, it is reasonable to assume the upper bounds on the time delays. Second, for any communication channel/communication protocol, where a received signal is used by the teleoperation system (i.e., not discarded) until a fresher (more recently sent) packet arrives, time delays cannot grow faster than the time. In other words, the time derivative of the delay is less than 1.

### 3. RADIAL BASIS FUNCTION NEURAL NETWORKS

The RBF NN can be considered as a two-layer network in which the hidden layer performs a fixed nonlinear transformation with no adjustable parameters, that is, the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly [29]. In this paper, the following RBF NN is used to approximate the continuous function $\Omega(x) : R^n \to R$,

$$\Omega_{nn}(x) = W^T S(x) \quad (9)$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \Omega_x \subset R^n$ is the input vector, and $W = [w_1, w_2, \ldots, w_n]^T \in R^n$ is the weight vector with the NN node number $n > 1$, and $S(x) = [s_1(x), \ldots, s_n(x)]^T$ with Gaussian functions $s_i(x)$, that is, $s_i(x) = \exp[-(x - \mu_i)^T (x - \mu_i)/\eta_i^2], i = 1, \ldots, n$, $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{id}]^T$ is the center of the receptive field and $\eta_i$ is the width of the Gaussian function.
It is well known that the NN is with universal approximation property [28], that is, if \( n \) is chosen sufficiently large, \( W^T S(x) \) can approximate any continuous function, \( \Omega(x) \), to any desired accuracy over a compact set \( \Omega_x \subset R^q \) to arbitrary any accuracy in the form of \( \Omega(x) = W^T S(x) + \epsilon(x) \), \( \forall x \in \Omega_x \subset R^q \) where \( W^* \) is the ideal constant weight vector, and \( \epsilon(x) \) is the approximation error that is bounded over the compact set, that is, \(|\epsilon(x)| \leq \epsilon^*, \forall x \in \Omega_x \) where \( \epsilon^* > 0 \) is an unknown constant. \( W^* \) is defined as the value of \( W \) that minimizes \(|\epsilon| \) for all \( x \in \Omega_x \subset R^q \), that is, \( W^* := \arg \min_{W \in R^n} \left( \sup_{x \in \Omega_x} |\Omega(x) - W^T S(x)| \right) \).

Thus, for the unknown nonlinear functions \( f_i(x_i), i = 1, \ldots, n \), we have the following approximation over the compact sets \( \Omega_x \subset R^q \)

\[
f_i(x_i) = W^*_i T S(x_i) + \epsilon_i(x_i), \forall x_i \in \Omega_x \subset R^q
\]

where \( S(x_i) \) is the Gaussian basis function with fixed center \( \mu_i \) and width \( \eta_i \), \( \epsilon_i(x_i) \) is the approximation error and \( W^* \) is an unknown constant parameter vector.

**Remark 3.1**

The optimal weight vector \( W^*_i \) in (10) is an artificial quantity required only for analytical purposes. Typically, \( W^*_i \) is chosen as the value of \( W_i \) that minimizes \( \epsilon_i(x_i) \) for all \( x_i \in \Xi_i \), where \( \Xi_i \subset R^i \) is a compact set, that is,

\[
W^*_i := \arg \min_{W_i \in R^n} \left\{ \sup_{x_i \in \Xi_i} \left| f_i(x_i) - W^T_i S(x_i) \right| \right\}
\]

On a compact region \( \Xi_i \subset R^i \)

\[
|\epsilon_i(x_i)| \leq \epsilon^*_i
\]

where \( \epsilon^*_i > 0 \) is unknown bound.

From the aforementioned analysis, we see that the system uncertainties are converted to the estimation of unknown parameters \( W^*_i \) and unknown bounds \( \epsilon^*_i \).

**Remark 3.2**

In [28], the stability results obtained are semi-global in the sense that, as long as the input variables \( x \) of the NNs remain within some pre-fixed compact set, \( \Omega_x \subset R^q \), where the compact set \( \Omega_x \) can be made as large as desired, there exists controller(s) with sufficiently large number of NN nodes such that all the signals in the closed-loop remain bounded. It should be noted that RBF NNs can be replaced by any linearly parameterized networks without any technical difficulty such as fuzzy systems, polynomial, splines, and wavelet networks.

### 4. MOTION SYNCHRONIZATION OF MASTER–SLAVE TELEOPERATION SYSTEM

Define the filtered tracking errors as

\[
r_m = \dot{q}_m + \Lambda_m e_m, \quad r_s = \dot{q}_s + \Lambda_s e_s
\]

where \( \Lambda_m \) and \( \Lambda_s \) are positive diagonal matrices. Moreover, it is easy to have the following computable signals:

\[
\dot{q}_{mr} = -\Lambda_m e_m, \quad \dot{q}_{sr} = -\Lambda_s e_s
\]

Because \( \dot{q}_m = -\Lambda_m e_m + r_m, \quad \dot{q}_s = -\Lambda_s e_s + r_s \) and \( \dot{q}_m = -\Lambda_m e_m + \dot{r}_m + \tilde{q}_m = -\Lambda_s e_s + \dot{r}_s + \tilde{q}_s \), let

\[
\mu_m = M_m \dot{q}_m + C_m \ddot{q}_m + G_m + f_m(\dot{q}_m) - F_h
\]

\[
\mu_s = M_s \dot{q}_s + C_s \ddot{q}_s + G_s + f_s(\dot{q}_s) - F_c
\]
Using (12), (13), (14) and (15), equations (1) and (2) become
\[ M_m(q_m) \ddot{r}_m = \tau_m - \mu_m, \quad M_s(q_s) \ddot{r}_s = \tau_s - \mu_s \] (16)
Define the following nonlinear feedback
\[ \tau_m = M_m(q)(U_m + M_m^{-1}(q)\mu_m) \] (17)
\[ \tau_s = M_s(q)(U_s + M_s^{-1}(q)\mu_s) \] (18)
where \( U_m \) and \( U_s \) are auxiliary control inputs to be defined later, therefore, the close-loop system for \( q_m \) and \( q_s \) subsystem becomes
\[ \dot{r} = U \] (19)
where \( r = [r_m^T, r_s^T]^T \), \( U = [U_m^T, U_s^T]^T \). Therefore, from (12), the master/slave robots state synchronize if the coordination errors and their derivatives approach the origin asymptotically.

It is easy to have
\[ \dot{e}_m(t) = \frac{d}{dt}[K_0 + K_h q_m(t) - K_e q_s(t - d_s(t))] = K_h[-\Lambda_m e_m(t) + r_m(t)] + K_e[\Lambda_s e_s(t - d_s(t)) - r_s(t - d_s(t))](1 - \dot{d}_s(t)) = -K_h \Lambda_m e_m(t) + K_e \Lambda_s e_s(t - d_s(t))(1 - \dot{d}_s(t)) - K_e r_s(t - d_s(t))(1 - \dot{d}_s(t)) + K_h r_m(t) \] (20)
\[ \dot{e}_s(t) = \frac{d}{dt}[-K_0 + K_e q_s(t) - K_h q_m(t - d_m(t))] = K_h[\Lambda_m e_m(t - d_m(t)) - r_m(t - d_s(t))][1 - \dot{d}_m(t)] - K_e \Lambda_s e_s(t) + K_e r_s(t) = -K_e \Lambda_s e_s(t) + K_h \Lambda_m e_m(t - d_m(t))[1 - \dot{d}_m(t)] - K_h r_m(t - d_m(t))[1 - \dot{d}_m(t)] + K_e r_s(t) \] (21)
Let \( e = [e_m^T, e_s^T]^T \), then we have
\[ \dot{e} = \begin{bmatrix} \dot{e}_m \\ \dot{e}_s \end{bmatrix} = \begin{bmatrix} -K_h \Lambda_m & 0 \\ 0 & -K_e \Lambda_s \end{bmatrix} \begin{bmatrix} e_m \\ e_s \end{bmatrix} + \begin{bmatrix} K_h r_m(t) - K_e r_s(t - d_s(t))(1 - \dot{d}_s(t)) \\ K_e r_s(t) - K_h r_m(t - d_m(t))(1 - \dot{d}_m(t)) \end{bmatrix} \] (22)
We could build up the following augmented system as
\[ \dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} \dot{e}_m \\ \dot{e}_s \\ \dot{r}_m \\ \dot{r}_s \end{bmatrix} = \begin{bmatrix} -K_h \Lambda_m & 0 & K_h & 0 \\ 0 & -K_e \Lambda_s & 0 & K_e \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_m \\ e_s \\ r_m \\ r_s \end{bmatrix} + \begin{bmatrix} K_h \Lambda_m[1 - \dot{d}_m(t)] & 0 & -K_e[1 - \dot{d}_2(t)] & 0 \\ 0 & K_h \Lambda_m[1 - \dot{d}_m(t)] & 0 & -K_e[1 - \dot{d}_m(t)] \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_m(t - d_m(t)) \\ e_s(t - d_s(t)) \\ r_m(t - d_m(t)) \\ r_s(t - d_s(t)) \end{bmatrix} \] (23)
which can be described by brief form

\[ \dot{X} = A_1 X + A_2 X(t - d_m(t), t - d_s(t)) + U \]  

(24)

with \( X = [X_1, X_2]^T, X_1 = [e_m^T, e_s^T], X_2 = [r_m^T, r_s^T], U = [0, 0, M_m^{-1}(q_m)(r_m - \mu_m), M_m^{-1}(q_s)(\tau_s - \mu_s)]^T, A_1, A_2 \in \mathbb{R}^{4n \times 4}, X, U \in \mathbb{R}^{4n \times 1}. \)

The proposed control for the system is given as

\[
U = \begin{bmatrix}
0 \\
0 \\
K_1 r_m(t) + K_2 r_s(t - d_s(t)) \\
K_3 r_m(t - d_m(t)) + K_4 r_s(t)
\end{bmatrix}
\]  

(25)

where \( K_i \in \mathbb{R}^{n \times n} \) is a diagonal positive.

The parameters \( M_j, C_j, G_j, f_j, j = m, s \) in dynamical models (1) and (2) are functions of physical parameters of teleoperation system like links masses, links lengths, moments of inertia and so on. The precise values of these parameters are difficult to acquire because of measuring errors, environment, and payloads variations. Therefore, it is assumed that actual value \( M_j, C_j, G_j, f_j \) can be separated as nominal parts denoted by \( M_j^0, C_j^0, G_j^0 \) and \( f_j^0 \) and uncertain parts denoted by \( \Delta M_j, \Delta C_j, \Delta G_j, \) and \( \Delta f_j \), respectively. These variables satisfy the following relationships:

\[
M_j = M_j^0 + \Delta M_j, \quad C_j = C_j^0 + \Delta C_j, \\
G_j = G_j^0 + \Delta G_j, \quad f_j = f_j^0 + \Delta f_j
\]

Suppose that dynamical models of robot manipulators are known precisely and unmodeled dynamics are excluded, that is, \( \Delta M_j, \Delta C_j, \Delta G_j, \) and \( \Delta f_j \) in (1) and (2) are all zeros. At this time, Equations (1) and (2) can be converted into the following nominal models:

\[
M_m^0(q_m)\ddot{q}_m + C_m^0(q_m, \dot{q}_m)\dot{q}_m + G_m^0(q_m) + f_m^0(\dot{q}_m) - F_h = \tau_m \quad (26)
\]

\[
M_s^0(q_s)\ddot{q}_s + C_s^0(q_s, \dot{q}_s)\dot{q}_s + G_s^0(q_s) + f_s^0(\dot{q}_s) - F_v = \tau_s \quad (27)
\]

Consider the coordinating torques as

\[
\tau_m^0 = M_m^0(K_1 r_m(t) + K_2 r_s(t - d_s(t))) + \mu_m \\
\tau_s^0 = M_s^0(K_3 r_m(t - d_m(t)) + K_4 r_s(t)) + \mu_s
\]  

(28)

(29)

The closed-loop system can be described as

\[
\dot{X} = \begin{bmatrix}
\dot{e}_m \\
\dot{e}_s \\
\dot{r}_m \\
\dot{r}_s
\end{bmatrix} = \begin{bmatrix}
-K_h \Lambda_m & 0 & K_h & 0 \\
0 & -K_e \Lambda_s & 0 & K_e \\
0 & 0 & K_1 & 0 \\
0 & 0 & 0 & K_4
\end{bmatrix} \begin{bmatrix}
e_m \\
e_s \\
r_m \\
r_s
\end{bmatrix} + \begin{bmatrix}
0 & K_e \Lambda_s[1 - \dot{d}_s(t)] & 0 & -K_e[1 - \dot{d}_s(t)]I \\
K_h \Lambda_m[1 - \dot{d}_m(t)] & 0 & -K_h[1 - \dot{d}_m(t)]I & 0 \\
0 & 0 & 0 & K_2 \\
0 & 0 & 0 & K_3
\end{bmatrix} \begin{bmatrix}
e_m(t - d_m(t)) \\
e_s(t - d_s(t)) \\
r_m(t - d_m(t)) \\
r_s(t - d_s(t))
\end{bmatrix}
\]

which can be decoupled into two subsystems as
X₁ subsystem:

\[
\dot{e} = A_{11} e + A_{13} e(t - d_m(t), t - d_s(t)) + A_{14} r(t - d_m(t), t - d_s(t))
\]

\[
A_{11} = \begin{bmatrix}
-K_h \Lambda_m & 0 \\
0 & -K_e \Lambda_s 
\end{bmatrix}, \quad A_{13} = \begin{bmatrix}
K_h & 0 \\
0 & K_e 
\end{bmatrix}
\]

\[
r(t - d_m(t), t - d_s(t)) = \begin{bmatrix}
r_1(t - d_m(t)) \\
r_2(t - d_s(t))
\end{bmatrix}
\]

\[
A_{12} = \begin{bmatrix}
0 & K_e \Lambda_s [1 - \dot{d}_s(t)] \\
K_h \Lambda_m [1 - \dot{d}_m(t)] & 0
\end{bmatrix}, \quad A_{14} = \begin{bmatrix}
0 & -K_e [1 - \dot{d}_s(t)] I \\
-K_h [1 - \dot{d}_m(t)] I & 0
\end{bmatrix}
\]

X₂ subsystem:

\[
\dot{r} = A_{21} r + A_{22} r(t - d_m(t), t - d_s(t))
\]

\[
A_{21} = \begin{bmatrix}
K_1 & 0 \\
0 & K_4
\end{bmatrix}, \quad A_{22} = \begin{bmatrix}
0 & K_2 \\
K_3 & 0
\end{bmatrix}
\]

One can imagine that model-based control is used to control nominal system and another NN based control adding to model-based control for uncertain system can be designed. Therefore, we propose the control law as

\[
\tau_m = \tau_m^0 + \Delta \tau_m
\]

\[
\tau_s = \tau_s^0 + \Delta \tau_s
\]

where \(\Delta \tau_m\) and \(\Delta \tau_s\) are used to compensate the dynamics uncertainty and will be defined later. The closed-loop can be described as

\[
\dot{X} = \begin{bmatrix}
\dot{e}_m \\
\dot{e}_s \\
\dot{r}_m \\
\dot{r}_s
\end{bmatrix} = \begin{bmatrix}
-K_h \Lambda_m & 0 & K_h & 0 \\
0 & -K_e \Lambda_s & 0 & K_e \\
0 & 0 & K_1 & 0 \\
0 & 0 & 0 & K_4
\end{bmatrix} \begin{bmatrix}
e_m \\
e_s \\
r_m \\
r_s
\end{bmatrix} + \begin{bmatrix}
0 & K_e \Lambda_s [1 - \dot{d}_s(t)] & 0 & -K_e [1 - \dot{d}_s(t)] I \\
K_h \Lambda_m [1 - \dot{d}_m(t)] & 0 & -K_h [1 - \dot{d}_m(t)] I & 0 \\
0 & 0 & 0 & K_2 \\
0 & 0 & 0 & K_3
\end{bmatrix} \begin{bmatrix}
e_m(t - d_m(t)) \\
e_s(t - d_s(t)) \\
r_m(t - d_m(t)) \\
r_s(t - d_s(t))
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & 0 \\
-U_m + M_m^{-1} \tau_m - M_m^{-1}(q_m) \mu_m \\
-U_s + M_s^{-1} \tau_s - M_s^{-1}(q_s) \mu_s
\end{bmatrix}
\]
Remark 4.1
It is easy to rewrite the last term as
\[-U_m + M_m^{-1}q_m - M_m^{-1}(q_m)(\mu_m)
= -U_m + M_m^{-1}(M_m^0U_m + \mu_m) + M_m^{-1}\Delta \tau_m - M_m^{-1}(q_m)(\mu_m + \Delta \mu_m)
= -U_m + M_m^{-1}M_m^0U_m + M_m^{-1}\Delta \tau_m - M_m^{-1}(q_m)\Delta \mu_m
= (M_m^{-1}M_m^0 - I)U_m + M_m^{-1}\Delta \tau_m - M_m^{-1}(q_m)\Delta \mu_m\]

Similarly, we can obtain
\[-U_s + M_s^{-1}x_s - M_s^{-1}(q_s)(\mu_s) = (M_s^{-1}M_s^0 - I)U_s + M_s^{-1}\Delta \tau_s - M_s^{-1}(q_s)\Delta \mu_s\]

Remark 4.2
Consider Remark 4.1, the closed-loop (34) can be decoupled into two subsystems as

\[\dot{X}_1 = A_{11}e + A_{12}e(t - d_m(t), t - d_s(t)) + A_{13}r + A_{14}r(t - d_m(t), t - d_s(t))\]

\[A_{11} = \begin{bmatrix}
-K_h \Lambda_m & 0 \\
0 & -K_e \Lambda_s 
\end{bmatrix},
A_{13} = \begin{bmatrix}
K_h & 0 \\
0 & K_e 
\end{bmatrix}
\]

\[e(t - d_m(t), t - d_s(t)) = \begin{bmatrix}
e_1(t - d_m(t)) \\
e_2(t - d_s(t))
\end{bmatrix}\]

\[A_{12} = \begin{bmatrix}
0 & K_e \Lambda_s[1 - \dot{d}_s(t)] \\
K_h \Lambda_m[1 - \dot{d}_m(t)] & 0
\end{bmatrix},
A_{14} = \begin{bmatrix}
0 & -K_e[1 - \dot{d}_s(t)]I \\
-K_h[1 - \dot{d}_m(t)]I & 0
\end{bmatrix}\]

\[\dot{X}_2 = A_{21}r + A_{22}r(t - d_m(t), t - d_s(t)) + \Xi\]

\[A_{21} = \begin{bmatrix}
K_1 & 0 \\
0 & K_4
\end{bmatrix},
A_{22} = \begin{bmatrix}
0 & K_2 \\
K_3 & 0
\end{bmatrix},
\Xi = \begin{bmatrix}
\Xi_m \\
\Xi_s
\end{bmatrix} = \begin{bmatrix}
(M_m^{-1}M_m^0 - I)U_m + M_m^{-1}\Delta \tau_m - M_m^{-1}(q_m)\Delta \mu_m \\
(M_s^{-1}M_s^0 - I)U_s + M_s^{-1}\Delta \tau_s - M_s^{-1}(q_s)\Delta \mu_s
\end{bmatrix}\]

Remark 4.2
From (35) and (36), we can see the nonlinear dynamics of the teleoperation system can be transformed into two subsystems: local master/slave dynamics control with the time-delay state information (36) and time-delay motion tracking (35). We first design the NN control with LMIs stability synthesis for the subsystem (36), and then develop the LMIs condition for the subsystem (35).

To prove the stability of this subsystem, some lemmas and assumptions will be introduced first as follows:

Assumption 4.1
The known positive parameters \(b_m, b_s, p_m, \) and \(p_s\) satisfy \(b_m \leq \lambda_{\min}(M_m^{-1})\) and \(\lambda_{\max}(P_m) \leq p_m, b_s \leq \lambda_{\min}(M_s^{-1}) \) and \(\lambda_{\max}(P_s) \leq p_s, \) that is \(x^Tb_mIx \leq x^TM_m^{-1}x, x^Tb_mIx \geq x^TP_mx, x^Tb_sIx \leq x^TM_s^{-1}x, x^Tb_sIx \geq x^TP_s^x\) with any vector \(x.\)
5. STABILITY ANALYSIS

5.1. \( X_2 \) Subsystem

From the \( X_2 \) subsystem model description earlier, the unsymmetrical delays are described by a stochastic model governed by Markov process, then the local dynamics system is casted into the framework of Markovian jump system. A general mathematical representation of a dynamical system with Markovian jumping parameters and mode-dependent time-varying delays is described by the following dynamics

\[
\dot{x}(t) = A_{21}(\eta_m, \eta_s)x(t) + A_{22}(\eta_m, \eta_s)x(t - d_m(t), t - d_s(t)) + \Xi
\]  

(37)

\( \eta_m(s) = \phi_1(s), \eta_{ms} = \eta_0, s \in [-2\bar{d}_m, 0] \)

\( \eta_s(s) = \phi_2(s), \eta_{ss} = \eta_0, s \in [-2\bar{d}_s, 0] \)

\[
r = \begin{bmatrix} r_m^T & r_s^T \end{bmatrix}^T
\]

\[
r(t - d_m(t), t - d_s(t)) = \begin{bmatrix} r_m(t - d_m(t)) \\ r_s(t - d_s(t)) \end{bmatrix}
\]

\[\Xi = \begin{bmatrix} \Xi_m \\ \Xi_s \end{bmatrix} \]

\[
(\begin{bmatrix} M_m^{-1}M_m^0 - I \\ M_s^{-1}M_s^0 - I \end{bmatrix} U_m + M_m^{-1}\Delta\tau_m - M_m^{-1}(q_m)\Delta\mu_m \\ (M_s^{-1}M_s^0 - I) U_s + M_s^{-1}\Delta\tau_s - M_s^{-1}(q_s)\Delta\mu_s)
\]

In order to cast the model into the framework of Markovian jump systems, we define a new process

\[
X_i \triangleq \begin{bmatrix} r_m(t + s, \eta_m), s \in [-2\bar{d}_m, 0] \\ r_s(t + s, \eta_s), s \in [-2\bar{d}_s, 0] \end{bmatrix}
\]

(38)

which takes values in the space

\[C_0 \triangleq \bigcup_{i \in S} C_{2n}[-2\bar{d}_m - 2\bar{d}_s, 0] \times \{i\}\]

Specifically, \((X_i, r_i)\) can be verified as a strong Markov process.

Definition 5.1 (Boukas et al. [35])

System (37) is said to be stochastically stable if there exist a constant \( \Gamma \) such that

\[
E \left\{ \int_0^\infty \|x(r_i, t)\|^2 dt \mid \phi(s), s \in [-2\bar{d}, 0], r_0 \right\} < \Gamma
\]

(39)

Theorem 5.1

The system (37) is exponentially stable in mean square if there exist \( 2n \times 2n \) positive-definite matrices \( P_i, Q_m, \) and \( Q_s, i \in S \) such that the following inequality holds for all \( i, l \in S \), the parameters in the above LMI will be defined in the following proof.

Proof

Consider the following Lyapunov–Krasovskii functional:

\[
V = \mathcal{V}_1 + \mathcal{V}_2 + \mathcal{V}_3 + \mathcal{V}_4 + \mathcal{V}_5
\]

(41)

where

\[ \mathcal{V}_1 = r^T(t)P_ir(t) + \sum_{j=m,s} \bar{g}_j^T \Omega_j^{-1} \bar{g}_j \]  
\[ \mathcal{V}_2 = \sum_{j=m,s} \int_{t-d_j,n_j(t)}^{t} r_j^T(s)Q_jr_j(s)ds \]  
\[ \mathcal{V}_3 = \sum_{j=m,s} \sigma \int_{-\tilde{d}_j}^{0} \int_{t+\theta}^{t} r_j^T(s)Q_jr_j(s)d\theta d\theta \]  
\[ \mathcal{V}_4 = \sum_{j=m,s} \int_{-\tilde{d}_j}^{0} \int_{t+\theta}^{t} \tilde{r}_j^T(s)R_j\hat{r}_j(s)d\theta d\theta \]  
\[ \mathcal{V}_5 = \sum_{j=m,s} \int_{t-\tilde{d}_j}^{t} r_j^T(s)Y_jr_j(s)ds \]

with \( \Omega_j = \text{diag}[\omega_{jk}] \) and \( P_i = P_i^T = \text{diag}[P_{mi}, P_{si}] > 0 \), \( Q_i = Q_i^T = \text{diag}[Q_{mi}, Q_{si}] > 0 \), \( R_i = R_i^T = \text{diag}[R_{mi}, R_{si}] > 0 \), \( Y_i = Y_i^T = \text{diag}[Y_{mi}, Y_{si}] > 0 \), \( j = m, s \), and to be determined. From the Leibniz–Newton formula, the following equations are true for any matrices \( N_1, N_2, S_1, S_2 \) with appropriate dimensions:

\[ 2 \left[ r_j^T(t)N_{1j} + r_j^T(t-d_j(t))N_{2j} \right] \left[ r_j(t) - r_j(t-d_j(t)) - \int_{t-d_j(t)}^{t} \tilde{r}_j(s)ds \right] = 0 \]

\[ 2 \left[ r_j^T(t)S_{1j} + r_j^T(t-d_j(t))S_{2j} \right] \left[ r_j(t-d_j(t)) - r_j(t-\tilde{d}_j) - \int_{t-\tilde{d}_j}^{t-d_j(t)} \tilde{r}_j(s)ds \right] = 0 \]

On the other hand, the following equations are also true:

\[ -\int_{t-\tilde{d}_j}^{t} \tilde{r}_j(s)Z_j\hat{r}_j(s)ds = -\int_{t-d_j(t)}^{t} \tilde{r}_j(s)Z_j\hat{r}_j(s)ds - \int_{t-\tilde{d}_j}^{t-d_j(t)} \tilde{r}_j(s)Z_j\hat{r}_j(s)ds \]

Applying the Markovian infinitesimal operator, we have

\[ \dot{\mathcal{V}}_1 = r^T P_i \dot{r} + \dot{r}^T P_i \dot{r} + 2\tilde{\Theta}^T \Omega^{-1} \hat{\Theta} \]

\[ = (A_{21i} \dot{r} + A_{22i} \tilde{r} + \Xi)P_i \dot{r} + \dot{r}^T P_i (A_{21i} \dot{r} + A_{22i} \tilde{r} + \Xi) + 2\tilde{\Theta}^T \Omega^{-1} \hat{\Theta} \]

\[ = r^T (A_{21i}^T P_i + P_i A_{21i}) \dot{r} + \dot{r}^T (A_{22i}^T P_i + P_i A_{22i}) \dot{r} + 2r^T P_i \Xi + 2\tilde{\Theta}^T \Omega^{-1} \hat{\Theta} \]

\[ = r^T (A_{21i}^T P_i + P_i A_{21i}) \dot{r} + \dot{r}^T (A_{22i}^T P_i + A_{22i}^T P_i) \dot{r} + 2 \left[ \begin{array}{c} \mathcal{T}_m \mathcal{T}_s^T \\ \mathcal{P}_m \mathcal{P}_s \end{array} \right] \left[ \begin{array}{c} \mathcal{T}_m \mathcal{T}_s^T \\ \mathcal{P}_m \mathcal{P}_s \end{array} \right] \Xi \]

\[ + 2\tilde{\Theta}^T \Omega^{-1} \hat{\Theta} \]

\[ \leq r^T (A_{21i}^T P_i + P_i A_{21i}) \dot{r} + \dot{r}^T (A_{22i}^T P_i + A_{22i}^T P_i) \dot{r} + 2\tilde{\Theta}^T \Omega^{-1} \hat{\Theta} + 2r^T P_m M_m \Delta \tau_m 
+ 2 \| P_m \mathcal{M}_m \mathcal{M}_m^0 - I \| \| U_m \| + 2 \| \| P_m \| \mathcal{M}_m^{-1} \| \| \Delta \mu_m \|
+ 2 \| P_s \| \mathcal{M}_s^{-1} \mathcal{M}_s^0 - I \| \| U_s \| + 2 \| \| P_s \| \mathcal{M}_s^{-1} \| \| \Delta \mu_s \|
+ 2 \| \mathcal{R} \| \mathcal{P}_s \| \mathcal{M}_s^{-1} \| \| \Delta \mu_s \|\]
The unknown continuous function \( \| P_{mi} \| \| M_{m}^{-1} M_{m}^{0} - I \| \| U_{m} \| + \| P_{mi} \| \| M_{m}^{-1} \| \| \Delta \mu_{m} \| \) can be approximated by NN to arbitrary any accuracy as
\[
\| P_{mi} \| \| M_{m}^{-1} M_{m}^{0} - I \| \| U_{m} \| + \| P_{mi} \| \| M_{m}^{-1} \| \| \Delta \mu_{m} \| = \Theta_{m}^{T} \Phi_{m}(Z_{m}) + \varepsilon_{m}(Z_{m})
\]
(51)
where the input vector \( Z_{m} = [\| \dot{q}_{mr} \|, \| \dot{q}_{m} \|, \| \dot{q}_{m} \|^2, 1, 1, \| \dot{q}_{m} \|, \| U_{m} \|]^{T} \in R^{7} \). Note that the input vector \( Z_{m} \) is composed of real elements (i.e., \( Z_{m} \in R^{7} \)). Moreover, \( \varepsilon_{m}(Z_{m}) \) is the approximation error satisfying \( |\varepsilon_{m}(Z_{m})| \leq \bar{\varepsilon}_{m} \), where \( \bar{\varepsilon}_{m} \) is an unknown positive constant; \( \Theta_{m}^{*} \in R^{n_{m}} \) are unknown ideal bounded constant weights; and \( \Phi_{m}(Z_{m}) \in R^{n_{m}} \) are the basis functions. By using \( \hat{\Theta}_{m} \) to approximate \( \Theta_{m}^{*} \), the error between the actual and the ideal NN can be expressed as
\[
\hat{\Theta}_{m}^{T} \Phi_{m}(Z_{m}) - \Theta_{m}^{*T} \Phi_{m}(Z_{m}) = \hat{\Theta}_{m}^{T} \Phi_{m}(Z_{m})
\]
(52)
where \( \hat{\Theta}_{m} = \hat{\Theta}_{m} - \Theta_{m}^{*} \). As \( \Theta_{m}^{*} \) is a constant vector, it is easy to obtain that
\[
\hat{\Theta}_{m}^{T} = \hat{\Theta}_{m}
\]
(53)
Similarly, the unknown continuous function \( \| P_{si} \| \| M_{s}^{-1} M_{s}^{0} - I \| \| U_{s} \| + \| P_{si} \| \| M_{s}^{-1} \| \| \Delta \mu_{s} \| \) can be approximated by NN to arbitrary any accuracy as
\[
\| P_{si} \| \| M_{s}^{-1} M_{s}^{0} - I \| \| U_{s} \| + \| P_{si} \| \| M_{s}^{-1} \| \| \Delta \mu_{s} \| = \Theta_{s}^{*T} \Phi_{s}(Z_{s}) + \varepsilon_{s}(Z_{s})
\]
(54)
where the input vector \( Z_{s} = [\| \dot{q}_{sr} \|, \| \dot{q}_{s} \|, \| \dot{q}_{s} \|^2, 1, 1, \| \dot{q}_{s} \|, \| U_{s} \|]^{T} \in R^{7} \). By using \( \hat{\Theta}_{s} \) to approximate \( \Theta_{s}^{*} \), the error between the actual and the ideal NN can be expressed as
\[
\hat{\Theta}_{s}^{T} \Phi_{s}(Z_{s}) - \Theta_{s}^{*T} \Phi_{s}(Z_{s}) = \hat{\Theta}_{s}^{T} \Phi_{s}(Z_{s})
\]
(55)
where \( \hat{\Theta}_{s} = \hat{\Theta}_{s} - \Theta_{s}^{*} \). As \( \Theta_{s}^{*} \) is a constant vector, it is easy to obtain that
\[
\hat{\Theta}_{s}^{T} = \hat{\Theta}_{s}
\]
(56)

Remark 5.1
If the NN \( \hat{\Theta}_{f}^{T} \Phi_{f}(Z_{f}) \), \( f = m, s \), are rich enough, that is if the centers of the localized Gaussian RBFs were evenly distributed to span the whole input space, and the number of tunable parameters is large enough to approximate the system dynamics, then from the universal approximation (10), the term can be approximated to zero with any degree of accuracy [28]. Because it can be smaller than any machine precision, it can be considered to be practically zero
\[
\varepsilon_{f}(Z_{f}) = 0
\]
(57)
We can design the control as
\[
\Delta \mu_{m} = - \frac{b_{m}}{p_{m}} \hat{\Theta}_{m}^{T} \Phi_{m}(Z_{m}) sgn(r_{m})
\]
(58)
\[
\Delta \mu_{s} = - \frac{b_{s}}{p_{s}} \hat{\Theta}_{s}^{T} \Phi_{s}(Z_{s}) sgn(r_{s})
\]
(59)
where the input vector \( Z_{m} = [\| \dot{q}_{mr} \|, \| \dot{q}_{m} \|, \| \dot{q}_{m} \|^2, 1, 1, \| \dot{q}_{m} \|, \| U_{m} \|] \in R^{7} \), and \( Z_{s} = [\| \dot{q}_{sr} \|, \| \dot{q}_{s} \|, \| \dot{q}_{s} \|^2, 1, 1, \| \dot{q}_{s} \|, \| U_{s} \|] \in R^{7} \), \( \Theta_{m}^{*T} \Phi_{m} \in R^{1 \times n_{m}} \) and \( \Theta_{s}^{*T} \Phi_{s} \in R^{1 \times n_{s}} \) are unknown optimal parameter vectors, and \( \Phi_{m}(Z_{m}) \in R^{n_{m}} \) and \( \Phi_{s}(Z_{s}) \in R^{n_{s}} \) are the Gaussian functions. Let \( \hat{\Theta}_{m} \in R^{n_{m}} \) and \( \hat{\Theta}_{s} \in R^{n_{s}} \) be estimation parameter vectors for \( \Theta_{m}^{*T} \) and \( \Theta_{s}^{*T} \), respectively, \( b_{m} \), \( b_{s} \), \( p_{m} \), \( p_{s} \) are four known positive parameters.

In developing control laws (58) and (59), the parameters \( \hat{\Theta}_{m} \) and \( \hat{\Theta}_{s} \) are estimations and cannot be obtained easily. Therefore, we choose the following adaptive law to update the estimations:
\[
\dot{\hat{\Theta}}_{m} = \Omega_{m} \| r_{m} \| \Phi_{m}
\]
(60)
\[
\dot{\hat{\Theta}}_{s} = \Omega_{s} \| r_{s} \| \Phi_{s}
\]
(61)
where \( \Omega_{m}, \Omega_{s} \) are positive diagonal.
Similarly, it is easy to have

\[
\mathcal{V}_i \leq r^T (A_{21i} P_i + P_i A_{21i}) r + \tilde{r}^T (A_{22i}^T P_i + A_{22i} P_i^T) r
\]
\[
+ 2\|r_m\|\Theta^T \Phi_m - 2\|r_m\|\hat{\Theta}^T \Phi_m + 2\Theta^T m_r \Phi_m
\]
\[
+ 2\|r_s\|\hat{\Theta}^T \Phi_s - 2\|r_s\|\Theta^T \Phi_s + 2\Theta_s \|r_s\|\Psi_s
\]
\[
\leq r^T (A_{21i} P_i + P_i A_{21i}) r + \tilde{r}^T (A_{22i}^T P_i + A_{22i} P_i^T) r
\]

(62)

Considering (43) and (44), we have

\[
\mathcal{V}_2 \leq r^T m(t) Q_m r_m(t) - (1 - \mu_m) r^T m(t - d_{1m}(t)) Q_m r_m(t - d_{1m}(t))
\]
\[
+ \sum_{l \neq i} \pi_{il} \int_{t-d_{1m}(t)}^t \tilde{r}^T m(s) Q_m r_m(s) ds + r^T s(t) Q_s r_s(t)
\]
\[
- (1 - \mu_s) r^T s(t - d_s(t)) Q_s r_s(t - d_s(t)) + \sum_{l \neq i} \pi_{il} \int_{t-d_s(t)}^t \tilde{r}^T s(s) Q_s r_s(s) ds
\]
\[
\leq \sum_{j=m,s} r^T (t) Q_j r_j(t) - (1 - \mu_j) r^T (t - d_j) Q_j r_j(t - d_j(t))
\]
\[
+ \sigma \int_{t-d_j}^t r^T (s) Q_j r_j(s) ds
\]

(63)

Similarly, it is easy to have

\[
\mathcal{V}_3 \leq \sum_{j=m,s} \sigma \tilde{r}^T j(t) Q_j r_j(t) - \sigma \int_{t-d_j}^t r^T (s) Q_j r_j(s) ds
\]

(64)

Similarly, it is easy to have

\[
\mathcal{V}_4 \leq \sum_{j=m,s} \tilde{r}^T j(t) R_j \tilde{r}_j(t) - \int_{t-d_j}^t \tilde{r}^T (s) R_j \tilde{r}_j(s) ds
\]

(65)

\[
\mathcal{V}_5 \leq \sum_{j=m,s} r^T (t) Y_j r_j - r^T (t - \tilde{d}_j) Y_j r^T (t - \tilde{d}_j)
\]

(66)

Combining the inequalities earlier, and considering (47), (48), and (49), we have

\[
\mathcal{V} \leq r^T (t) \left( A_{21i} P_i + P_i A_{21i} + \sum_{l=1}^N \pi_{il} P_l \right) r(t) + 2r^T (t) P_i A_{22i} r(t - d_{1m}(t), t - d_{1m}(t))
\]
\[
+ r^T m(t) Q_m (1 + \sigma \tilde{r}_m) r_m(t) - (1 - \mu_m) r^T m(t - d_{1m}(t)) Q_m r_m(t - d_{1m}(t))
\]
\[
+ r^T s(t) Q_s (1 + \sigma \tilde{r}_s) r_s(t) - (1 - \mu_s) r^T s(t - d_s(t)) Q_s r_s(t - d_s(t))
\]
\[
+ r^T m(t) Y_m r_m - r^T m(t - \tilde{d}_m) Y_m r_m(t - \tilde{d}_m) + r^T s(t) Y_s r_s - r^T s(t - \tilde{d}_s) Y_s r^T s(t - \tilde{d}_s)
\]
\[
+ \tilde{d}_m r^T m(t) R_m \tilde{r}_m(t) - \int_{t-d_{1m}}^t \tilde{r}^T m(s) R_m \tilde{r}_m(s) ds + \tilde{d}_s r^T s(t) R_s \tilde{r}_s(t) - \int_{t-d_{1m}}^t \tilde{r}^T s(s) R_s \tilde{r}_s(s) ds
\]
\[
+ 2 \sum_{j=m,s} \left[ r^T (t) N_{1j} + r^T (t - d_j(t)) N_{2j} \right] \left[ r_j(t) - r_j(t - d_j(t)) - \int_{t-d_j(t)}^t \tilde{r}_j(s) ds \right]
\]
\[
+ 2 \sum_{j=m,s} \left[ r^T (t) S_{1j} + r^T (t - d_j(t)) S_{2j} \right] \left[ r_j(t) - r_j(t - d_j(t)) - r_j(t - \tilde{d}_j) - \int_{t-d_j(t)}^t \tilde{r}_j(s) ds \right]
\]
\textbf{5.2. } \textit{X}_1 \textbf{Subsystem}

In the former subsection, we have proved that system (37) is exponentially stable in the mean square, which implies that \( r(t) \) and \( r(t - d_m(t)), t - d_s(t) \) are both integratable in mean square, that is, \( r(t), r(t - d_m(t)), t - d_s(t) \in L_2[0, \infty) \). From previous stability proof of subsystem \( X_2 \), we know that the signals \( r_m(t), r_s(t) \in L_\infty[0, \infty) \), and from the definition of \( r_m(t) \) and \( r_s(t) \), we know \( \dot{\varepsilon}_m, \dot{\varepsilon}_s, q_m \), and \( q_s \) are bounded. Similar to \( X_2 \) subsystem, the time-varying delays in \( X_1 \) subsystem are described by a stochastic model governed by a Markov process, so that the system could be casted into the framework of Markovian jump systems.
In order to cast the model into the framework of Markovian jump systems, we define a new process

\[
X_t \triangleq \begin{bmatrix} e_m(t + s, \eta_m), s \in [-2\tilde{d}_m, 0] \\ e_s(t + s, \eta_s), s \in [-2\tilde{d}_s, 0] \end{bmatrix}
\]

which takes values in the space

\[
C_0 \triangleq \bigcup_{i \in S} C_{2n}^1[-2\tilde{d}_m - 2\tilde{d}_s, 0] \times \{i\}
\]

Specifically, \((X_t, r_t)\) can be verified as a strong Markov process.

Consider the following subsystem as

\[
\dot{e} = A_{11}(\eta_m, \eta_s)e(t) + A_{13}(\eta_m, \eta_s)r + A_{12}(\eta_m, \eta_s)e(t - d_m(t), t - d_s(t)) + A_{14}(\eta_m, \eta_s)r(t - d_m(t), t - d_s(t))
\]

where \(A_{11}(\eta_m, \eta_s) = \begin{bmatrix} -\Lambda_m K_h & 0 \\ 0 & -\Lambda_s K_e \end{bmatrix}, A_{12}(\eta_m, \eta_s) = \begin{bmatrix} 0 & K_e \Lambda_s \\ K_h \Lambda_m & 0 \end{bmatrix}\) and

\[
A_{13}(\eta_m, \eta_s) = \begin{bmatrix} \hat{K}_h & 0 \\ 0 & \hat{K}_e \end{bmatrix}, r(t - d_m(t), t - d_s(t)) = \begin{bmatrix} r_m(t - d_m(t)) \\ r_s(t - d_s(t)) \end{bmatrix}, A_{14}(\eta_m, \eta_s) = \begin{bmatrix} 0 & -K_e(1 - \hat{d}_s(t))I \\ -K_h(1 - \hat{d}_m(t))I & 0 \end{bmatrix}.
\]

Define a variable

\[
\xi(t) \triangleq \begin{bmatrix} e^T(t), \hat{e}^T(t), r^T(t), \hat{r}^T(t), r^T(t - d_m(t)) \end{bmatrix}^T
\]

with \(\hat{r}(t) = r(t - d_m(t), t - d_s(t)), \hat{e}(t) = e(t - d_m(t), t - d_s(t))\), we know that \(\xi(t) \in L_2\).

Noting that \(|\hat{d}_i(t)| < \tilde{\mu}\) where \(\tilde{\mu} = \max(\mu_1, \mu_2)\), it is obvious that the uncertainties satisfy the following form

\[
A_{12} = D_1 F_1(t) E_1, \quad A_{14} = D_2 F_2(t) E_2
\]

where \(D_1, E_1, D_2,\) and \(E_2\) are known constant matrices with compatible dimensions, and \(F_1(t)\) and \(F_2(t)\) are time-varying matrices representing the parameter uncertainties and satisfying

\[
F_1^T(t) F_1(t) \leq I, \quad F_2^T(t) F_2(t) \leq I
\]

Lemma 5.1 ([36])

For any matrices \(D, F, E\) with compatible dimensions and positive scalar \(\varepsilon\), we have the following inequality

\[
DF(t)E + ET F(t)^T D^T \leq \varepsilon DD^T + \varepsilon^{-1} E^TE
\]

where the matrix \(F(t)\) satisfies \(F^T(t) F(t) \leq I\).

Theorem 5.2

If there exists positive matrices \(R, L = \text{diag}[L_m, L_s], S = \text{diag}[S_m, S_s]\) and \(Y\), the positive scalars \(\varepsilon_1\) and \(\varepsilon_2\), \(M = [M]_{5 \times 5} \geq 0\), such that the following LMIs hold
Consider the following Lyapunov–Krasovskii functionals:

\[
\Omega_1 \quad RD_1 \quad RD_2 \quad dM_{12} \quad \Omega_3 \quad dM_{14} \quad dM_{15} \\
* \quad -\xi_1^{-1}I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
* \quad * \quad -\xi_2^{-1}I \quad 0 \quad 0 \quad 0 \quad 0 \\
* \quad * \quad \Omega_2 \quad dM_{23} \quad dM_{24} \quad dM_{25} \\
* \quad * \quad * \quad * \quad \Upsilon \quad dM_{34} \quad \Omega_6 \\
* \quad * \quad * \quad * \quad * \quad \Omega_4 \quad dM_{45} \\
* \quad * \quad * \quad * \quad * \quad \Omega_5
\]

\[
< 0 \quad (74)
\]

\[
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & 0 \\
* & M_{22} & M_{23} & M_{24} & M_{25} & 0 \\
* & * & M_{33} & M_{34} & M_{35} & -Q_1 \\
* & * & * & M_{44} & M_{45} & 0 \\
* & * & * & * & M_{55} & Q_2 \\
* & * & * & * & * & 2Q_2
\end{bmatrix} \geq 0 \quad (75)
\]

where \( \Upsilon = -2Q_1 + dM_{33} + (1 + d\sigma)(Y + S) \), \( \Omega_1 = A_{111}^T R_i + R_i A_{111} + L + dM_{11} + d\sigma L \), \( \Omega_2 = -(1-\mu)L + \xi_1^{-1}E_1^T E_1 + dM_{22}, \Omega_3 = dM_{13} + R_i A_{13}, \Omega_4 = -(1-\mu)S + \xi_2^{-1}E_2^T E_2 + dM_{44}, \) and \( \Omega_5 = -(1-\mu)Y + dM_{55}, \Omega_6 = dM_{35} + Q_1, \) and \( I \) denotes the identity matrix of appropriate dimension, then we conclude that the system (71) is stochastically stable.

**Proof**

Consider the following Lyapunov–Krasovskii functionals:

\[
V = V_1 + V_2 + V_3 + V_4 + V_5 \quad (76)
\]

where

\[
V_1 = e^T(t)R(r_t)e(t) \quad (77)
\]

\[
V_2 = \int_{t-dm(t)}^{t} \int_{t-ds(t)}^{t} e_m^T(s) L_m e_m(s) ds + \int_{t-dm(t)}^{t} \int_{t-ds(t)}^{t} e_s^T(s) L_s e_s(s) ds \quad (78)
\]

\[
V_3 = \int_{t-dm(t)}^{t} \int_{t-ds(t)}^{t} r_m^T(s) S_m r_m(s) ds + \int_{t-dm(t)}^{t} \int_{t-ds(t)}^{t} r_s^T(s) S_s r_s(s) ds \quad (79)
\]

\[
V_4 = \int_{t-dm(t)}^{t} r^T(s) Y r(s) ds \quad (80)
\]

\[
V_5 = \sigma \int_{-\bar{d}_m}^{0} \int_{t+\theta}^{t} r_m^T(s) S_m r_m(s) ds d\theta + \sigma \int_{-\bar{d}_m}^{0} \int_{t+\theta}^{t} r_s^T(s) S_s r_s(s) ds d\theta + \sigma \int_{-\bar{d}_m}^{0} \int_{t+\theta}^{t} e_m^T(s) L_m e_m(s) ds d\theta + \sigma \int_{-\bar{d}_m}^{0} \int_{t+\theta}^{t} e_s^T(s) L_s e_s(s) ds d\theta + \sigma \int_{-\bar{d}_m}^{0} \int_{t+\theta}^{t} r^T(s) Y r(s) ds d\theta \quad (81)
\]

Applying the Markovian infinitesimal operator to \( V_1 \), which is defined in [37], we have

\[
\mathcal{L}V_1 = e^T(t) \left(A_{111}^T R_i + R_i A_{111} + \sum_{l=1}^{N} \pi_{l} R_i \right) e(t) + 2e^T(t) R_i A_{13}^T r(t) + 2e^T(t) R_i A_{12} \bar{v}(t) + 2e^T(t) A_{14i} \bar{v}(t) \quad (82)
\]
Consider the form of $A_{12i}$ and $A_{14i}$, we can get following inequalities with (73)
\[
2e^T(t)R_1A_{12i}\bar{e}(t) \leq \varepsilon_1e^T(t)R_1D_1A_{12i}^T R_1 e(t) + \varepsilon_1^{-1}\bar{e}^T(t)E_1^T E_1 \bar{e}(t) \\
2e^T(t)R_1A_{14i}\bar{e}(t) \leq \varepsilon_2e^T(t)R_1D_2A_{14i}^T R_1 e(t) + \varepsilon_2^{-1}\bar{e}^T(t)E_2^T E_2 \bar{e}(t)
\]
(83)
(84)
Substituting (83) and (84) into (82), we have
\[
\mathcal{L}V_1 \leq e^T(t) \left( A_{11i}^T R_i + R_i A_{11i} + \sum_{l=1}^N \pi_{ll} R_l \right) e(t) \\
+ 2e^T(t)R_1A_{13i}r(t) + \varepsilon_1e^T(t)R_1D_1A_{13i}^T R_1 e(t) \\
+ \varepsilon_1^{-1}\bar{e}^T(t)E_1^T E_1 \bar{e}(t) + \varepsilon_2e^T(t)R_1D_2A_{14i}^T R_1 e(t) \\
+ \varepsilon_2^{-1}\bar{e}^T(t)E_2^T E_2 \bar{e}(t)
\]
(85)
Applying the Markovian infinitesimal operator to $V_2$, let $\mu = \max\{\mu_m, \mu_s\}$, we have
\[
\mathcal{L}V_2 \leq e^T_m(t)L_m e_m(t) + \sum_{j=1}^N \pi_{ij} \int_{t-d_m(t)}^t e^T_m(s)L_m e_m(s)ds \\
- (1-\mu_m)e^T(t-d_m(t))L_m e(t-d_m(t)) \\
+ e^T_s(t)L_s e_s(t) + \sum_{j=1}^N \pi_{ij} \int_{t-d_s(t)}^t e^T_s(s)L_s e_s(s)ds \\
- (1-\mu_s)e^T(t-d_s(t))L_s e(t-d_s(t)) \\
\leq e^T(t)Le(t) - (1-\mu)\bar{e}^T(t) \bar{e}(t) + \sum_{j=m,s} \sigma \int_{t-d_j(t)}^t e^T_j(s)L_j e_j(s)ds
\]
(86)
Similarly, we have
\[
\mathcal{L}V_3 \leq r^T(t)Sr(t) - (1-\mu)\bar{r}^T(t) \bar{r}(t) + \sum_{j=m,s} \sigma \int_{t-d_j(t)}^t r^T_j(s)S_j r_j(s)ds
\]
(87)
\[
\mathcal{L}V_4 \leq r^T(t)Yr(t) - (1-\mu)r^T(t-d_m(t))Yr(t-d_m(t)) + \sigma \int_{t-d_m(t)}^t r^T(s)Yr(s)ds
\]
(88)
\[
\mathcal{L}V_5 \leq \sum_{j=m,s} \sigma \tilde{d}_j r_j^T(t)S_j r_j(t) - \sigma \int_{t-\tilde{d}_j}^t r^T_j(s)S_j r_j(s)ds \\
+ \sum_{j=m,s} \sigma \tilde{d}_j e_j^T(t)L_j e_j(t) - \sigma \int_{t-\tilde{d}_j}^t e^T_j(s)L_j e_j(s)ds \\
+ \sigma \tilde{d}_m r^T(t)Yr(t) - \sigma \int_{t-\tilde{d}_m}^t r^T(s)Yr(s)ds
\]
(89)
Consider Assumption 2.1, for any semi-positive definite matrix $M_{5\times5} \succeq 0$, the following holds
\[
d\hat{\xi}^T(t)M\hat{\xi}(t) - \int_{t-d_m(t)}^t \hat{\xi}^T(s)M\hat{\xi}(s)ds \succeq 0,
\]
where $\hat{\xi}^T(t) = (e^T(t), \bar{e}^T(t), r^T(t), \bar{r}^T(t))$. On the other hand, according to the Newton–Leibnitz formula, it follows that
\[
r(t) - r(t-d_m(t)) = \int_{t-d_m(t)}^t \dot{r}(s)ds.
\]
Then, for any appropriately dimensioned matrices $Q_1$ and
\( Q_2 \), we can obtain
\[
(Q_1 r(t) - Q_2 \int_{t-d_m(t)}^t \dot{r}(s) \text{d}s) \dot{r}(t) = 0.
\]
Combining the previous two equations and the derivative (85)–(89), we have
\[
\mathcal{L}V = \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3 + \mathcal{L}V_4 + \mathcal{L}V_5
\]
\[
\leq e^T(t) \left( A^T_{111} R_i + R_i A_{111} + \sum_{i=1}^N \pi_{ii} R_i \right) e(t) + 2e^T(t) RA_{131} r(t)
\]
\[
+ \varepsilon_1 e^T(t) R_i D_1 D_i^T R_i e(T) + \varepsilon_1^{-1} \bar{e}^T(t) E_1^T E_1 \bar{e}(t)
\]
\[
+ \varepsilon_2 e^T(t) R_i D_2 D_i^T R_i e(T) + \varepsilon_2^{-1} \bar{e}^T(t) E_2^T E_2 \bar{e}(t)
\]
\[
+ e^T(t) L e(t) - (1 - \mu) \bar{e}^T(t) L \bar{e}(t) + \sum_{j=m,s} \sigma \int_{t-d_j(t)}^t e_j^T(s) L_j e_j(s) \text{d}s
\]
\[
+ r^T(t) S r(t) - (1 - \mu) \bar{r}^T(t) S \bar{r}(t) + \sum_{j=m,s} \sigma \int_{t-d_j(t)}^t r_j^T(s) S_j r_j(s) \text{d}s
\]
\[
+ \sum_{j=m,s} \sigma \bar{d}_j r_j^T(t) S_j r_j(t) - \sigma \int_{t-d_j}^t r_j^T(s) S_j r_j(s) \text{d}s
\]
\[
+ \sum_{j=m,s} \sigma \bar{d}_j e_j^T(t) L_j e_j(t) - \sigma \int_{t-d_j}^t e_j^T(s) L_j e_j(s) \text{d}s
\]
\[
+ \sigma \bar{d}_m r^T(t) Y r(t) - \sigma \int_{t-d_m(t)}^t r^T(s) Y r(s) \text{d}s
\]
\[
- 2(Q_1 r(t) - Q_2 \int_{t-d_m(t)}^t \dot{r}(s) \text{d}s) \dot{r}(t)
\]
\[
- r(t - d_m(t)) - \int_{t-d_m(t)}^t \dot{r}(s) \text{d}s
\]
\[
+ d \xi^T(t) M \xi(t) - \int_{t-d_m(t)}^t \xi^T(t) M \xi(t) \text{d}s
\]
\[
= \xi^T(t) \Pi_1 \xi(t) - \int_{t-d_m(t)}^t \xi^T(t,s) \Sigma \xi(t,s) \text{d}s
\]
\[
\leq \xi^T(t) \Pi_1 \xi(t)
\]
where \( \xi(t,s) = (\xi^T(t), \dot{\xi}^T(s))^T \), and
\[
\Pi_i = \begin{bmatrix}
\Pi_{11} & d M_{12} & \Pi_{13} & d M_{14} & d M_{15} \\
* & \Pi_{22} & d M_{23} & d M_{24} & d M_{25} \\
* & * & \Pi_{33} & d M_{34} & d M_{35} \\
* & * & * & \Pi_{44} & d M_{45} \\
* & * & * & * & \Pi_{55}
\end{bmatrix}
\]
\[
< 0
\]
(90)
where \( \Pi_{11} = \Psi_i + d M_{11}, \Pi_{22} = -(1 - \mu) L + \varepsilon_1^{-1} E_1^T E_1 + d M_{22}, \Pi_{33} = -2 Q_1 + d M_{33} + (1 + d \sigma)(Y + S), \Pi_{44} = -(1 - \mu) S + \varepsilon_2^{-1} E_2^T E_2 + d M_{44}, \Pi_{55} = -(1 - \mu) Y + d M_{55}, \)
The dynamics of two degree of freedom robotic manipulators are described as

\[ \Pi_{13} = dM_{13} + RA_{13}, \quad \Pi_{35} = dM_{35} + Q_1 \text{ and } \Psi_i = A^T_{11i}R_i + R_iA_{11i} + \varepsilon_1 R_iD_iD_i^TR_i + \varepsilon_2 R_iD_iD_i^TR_i + L_i(1 + d\sigma). \]

We assume that there exists a pair of master and slave manipulators in the teleoperation system. The simulations are performed on two degree of freedom robotic manipulators shown in Figure 2.

Remark 5.2

The proposed control can be extended for the case with unknown impedance parameters, if there are the force/torque sensor mounted in the joint space, we can implement the proposed approach in [38] to estimate the impedance parameters even if there exist time delays in the teleoperation communication channels.

6. SIMULATION STUDIES

The simulations are performed on two degree of freedom robotic manipulators shown in Figure 2. We assume that there exists a pair of master and slave manipulators in the teleoperation system. The dynamics of two degree of freedom robotic manipulators are described as

\[
\Sigma = \begin{bmatrix}
M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & 0 \\
* & M_{22} & M_{23} & M_{24} & M_{25} & 0 \\
* & * & M_{33} & M_{34} & M_{35} & -Q_1 \\
* & * & * & M_{44} & M_{45} & 0 \\
* & * & * & * & M_{55} & Q_2 \\
* & * & * & * & 2Q_2 & 0
\end{bmatrix} \geq 0
\]

We choose

\[ \alpha = \max_{i \in S} \lambda_{\max}(\Pi_i) \]

Obviously, \( \alpha < 0 \).

Then we have

\[ \mathcal{L}V(X_t, \tau_t, t) \leq \alpha \| \xi \|^2 \leq \alpha \| e(t) \|^2 \]  

(91)

From the Dynkin’s formula

\[
E\{V(x(t), r_t, t) - V(x_0, r_0, 0)\} \leq \alpha E \left\{ \int_0^t \| \xi(s) \|^2 ds \right\}
\]

\[ E \left\{ \int_0^t \| \xi(s) \|^2 ds \right\} \leq (-\alpha)^{-1} V(x_0, r_0, 0) \]

Let \( t \to +\infty \), then we have

\[ \lim_{t \to +\infty} E \left\{ \int_0^t \| \xi(s) \|^2 ds \right\} \leq (-\alpha)^{-1} V(x_0, r_0, 0) \]

It is noted that the stochastic stability is obtained from definition (5.1) if LMIs (74) and (75) are true. Therefore, the proof is completed.

Remark 5.2

The proposed control can be extended for the case with unknown impedance parameters, if there are the force/torque sensor mounted in the joint space, we can implement the proposed approach in [38] to estimate the impedance parameters even if there exist time delays in the teleoperation communication channels.

The simulations are performed on two degree of freedom robotic manipulators shown in Figure 2. We assume that there exists a pair of master and slave manipulators in the teleoperation system. The dynamics of two degree of freedom robotic manipulators are described as

\[
M_m(q_m) \ddot{q}_m + f_m = \tau_m + F_{\text{e}}, \quad M_s(q_s) \ddot{q}_s + G_s(q_s) + f_s = \tau_s + F_{\text{e}}, \quad \text{where} \quad M_j = \text{diag}[M_{j11}, M_{j22}], \quad G_j(q) = [G_{j1}, G_{j2}]^T, \quad M_{j11} = m_j l_{j1}^2 + m_j l_{j1} l_{j2} + m_j l_{j2} l_{j1} + m_j l_{j2} l_{j2} \sin(q_{j1} + q_{j2} \sin(q_{j1})), \quad G_{j2} = 0, \quad j = m, s. \quad \text{The human force} \quad F_{\text{h}} \quad \text{and the environmental force} \quad F_{\text{e}} \quad \text{are defined in (3) and (4), the frictions} \quad f_m \quad \text{and} \quad f_s \quad \text{are considered using Coulomb and Viscous model [39] and defined as} \quad f_j = \alpha_1 \text{sign}(\dot{q}_j) + \alpha_2 \dot{q}_j, \quad j = m, s. \quad \text{In the simulation, we choose the physical parameters} \quad m_{m1} = 0.6 \text{ kg,} \quad m_{s1} = 0.8 \text{ kg,} \quad m_{m2} = 0.6 \text{ kg,} \quad m_{s2} = 0.8 \text{ kg,} \quad l_{m1} = 0.5 \text{ m,} \quad l_{s1} = 0.9 \text{ m,} \quad l_{m2} = 0.9 \text{ m,} \quad l_{s2} = 0.7 \text{ m,} \quad l_{cm1} = 0.2 \text{ m,} \quad l_{cm2} = 0.5 \text{ m,} \quad l_{cs1} = 0.3 \text{ m,} \quad l_{cs2} = 0.3 \text{ m,} \quad I_1 = 2 \text{ Nm}^2, \quad I_2 = 4 \text{ Nm}^2, \quad g = 9.8 \text{ m/s}^2, \quad \alpha_1 = 0.05 \quad \text{and} \quad \alpha_2 = 0.1. \]
Remark 6.1
In this paper, bilateral teleoperation is casted into the framework of Markovian jump system with mode-dependent Markovian jump parameters. In our simulation, we assume that Markovian jump system has two modes, namely mode 1 and mode 2, and the mode-dependent parameters of the two different modes in the simulation are distinguished by index 1 and 2.

The feedback gain parameters in (25) are chosen as

\[
K_{11} = \begin{bmatrix} -4.3 & -0.1 \\ 0.3 & -7.9 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} -5 & -0.2 \\ 0.1 & -5 \end{bmatrix}, \quad K_{21} = \begin{bmatrix} -0.5 & 0.4 \\ 0.1 & -0.2 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, \quad K_{31} = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & -0.2 \end{bmatrix},
\]

\[
K_{32} = \begin{bmatrix} 0.3 & 0.4 \\ 0.1 & -0.1 \end{bmatrix}, \quad K_{41} = \begin{bmatrix} -4.8 & 0.2 \\ 0.1 & -3.8 \end{bmatrix}, \quad K_{42} = \begin{bmatrix} -2.8 & 0.1 \\ -0.3 & -4.5 \end{bmatrix}.
\]

These chosen feedback gain should be verified by the LMIs, if we can obtain the LMIs solution, that is, these gain setting can be used in the controller. Because our purpose is to verify the effectiveness of the proposed control, in this simulation example, we provide the transition rate for the Markov process as \( \Pi = [-16, 16; 8, -8] \), and it might be feasible that we can choose other transition rate matrices. The time delays in the simulation are chosen as \( d_{11}(t) = 0.6 \sin^2(t) \), \( d_{12}(t) = 0.4 \cos^2(t) \), \( d_{21}(t) = 0.5 \cos^2(t) \), \( d_{22}(t) = 0.5 \sin^2(t) \) and the upper and lower bounds of the values, and the derivatives of the time-varying delays are \( \dot{d}_m = 0.6 \), \( \dot{\mu}_m = 0.6 \), \( \dot{d}_s = 0.5 \), and \( \dot{\mu}_s = 0.5 \) respectively. \( \Lambda_{m1} = \Lambda_{s1} = \text{diag}[4.1, 4.4], \quad \Lambda_{m2} = \Lambda_{s2} = \text{diag}[2.3, 2.8], \quad K_h = K_e = 2.5 \text{ N/degree}, \quad C_h = C_e = 0.2 \text{ N/degree/s}, \quad K_{h0} = 0.25 \text{ N/m}, \quad \text{and} \quad K_{e0} = 0.5 \text{ N/m}.\)

From the system parameters chosen earlier, we can solve (40) using the LMI toolbox in the MATLAB and obtain as

\[
P_1 = \begin{bmatrix} 8.7390 & 0.0401 & 0 & 0 \\ 0.0401 & 6.8291 & 0 & 0 \\ 0 & 0 & 10.2404 & 0.0346 \\ 0 & 0 & 0.0346 & 9.8030 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 28.3095 & 0.0924 & 0 & 0 \\ 0.0924 & 33.7209 & 0 & 0 \\ 0 & 0 & 26.5053 & -0.9594 \\ 0 & 0 & -0.9594 & 27.5050 \end{bmatrix},
\]

\[
Q = \begin{bmatrix} 8.1878 & -0.0646 & 0 & 0 \\ -0.0646 & 8.4187 & 0 & 0 \\ 0 & 0 & 8.1757 & 0.0606 \\ 0 & 0 & 0.0606 & 8.5465 \end{bmatrix},
\]

\[
R = \begin{bmatrix} 2.4455 & 0.0251 & 0 & 0 \\ 0.0251 & 1.4537 & 0 & 0 \\ 0 & 0 & 3.6028 & 0.0709 \\ 0 & 0 & 0.0709 & 3.2979 \end{bmatrix}, \quad Y = \begin{bmatrix} 8.1878 & -0.0646 & 0 & 0 \\ -0.0646 & 8.4187 & 0 & 0 \\ 0 & 0 & 8.1757 & 0.0606 \\ 0 & 0 & 0.0606 & 8.5465 \end{bmatrix}.
\]

Figure 2. A two degree of freedom (2DOF) robotic manipulator.

We can choose $p_m = p_s = 18$, which is obviously greater than the maximum eigenvalue of $P$. We choose other parameters in this simulation as $\Omega_m = \Omega_s = \text{diag}[3.0]$, $b_m = b_s = 0.7$. The initial states for $X_2$ are assumed to be $r(t) = [0.8 \sin t, 0.7 \cos t, 0.8 \sin^2 t + 2, 0.7 \cos^2 t + 2]^T$. The state trajectories of subsystem $X_2$ are shown in Figure 3.

Consider the form of $A_{12i}$, $A_{14i}$, and the inequalities (73), we can define the parameters $D_1$, $D_2$, $E_1$, and $E_2$ as follows:

$$D_1 = \begin{bmatrix} 0 & K_e \\ K_h \Lambda_m & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 & -K_e \\ -K_h & 0 \end{bmatrix},$$

and $E_1 = E_2 = (\mu + 1)/I$. Using the LMI toolbox to solve LMI of $X_1$ system (74) and (75), we have $R_1 = \text{diag}[0.2076, 0.1927, 0.2076, 0.1927] \times 10^{-4}$, $R_2 = \text{diag}[0.1564, 0.1438, 0.1564, 0.1438] \times 10^{-4}$, $L = \text{diag}[0.1992, 0.2010, 0.1992, 0.2010] \times 10^{-3}$, $S = \text{diag}[0.1966, 0.1967, 0.1966, 0.1967] \times 10^{-3}$, $Y = \text{diag}[0.1992, 0.2010, 0.1992, 0.2010] \times 10^{-3}$, $Q_1 = \text{diag}[0.0011, 0.0011, 0.0011, 0.0011]$, and $Q_2 = \text{diag}[0.0012, 0.0012, 0.0012, 0.0012]$. The initial states of $X_1$ are $X_1(t) = [0.8 \sin t, 0.9 \cos t, 0.8 \sin^2 t + 2, 0.8 \cos^2 t + 2]^T$. The trajectories of $X_1$ subsystem are shown in Figure 4.

The adaptive NN control without any knowledge of system dynamics under the random noise inputting to the controllers. The input vectors is $Z_m \in R^7$. NNs $\hat{\Theta}_m^T \Phi_m(Z_m)$ contains 35 nodes, with centers $\mu_1(l = 1, \ldots, l_1)$ evenly spaced in $[-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0]$, NNs $\hat{\Theta}_s^T \Phi_s(Z_s)$ contain 35 nodes, with centers $\mu_1(l = 1, \ldots, l_2)$ evenly spaced in $Z_s \in R^7$, $[-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0]$. The synchronization performances are listed from Figures 5–9. Figure 5 shows the joint position trajectories of both master and slave robots. Input torques for the master and slave robots are shown in Figure 6.
in Figure 7. As we can see from Figure 4, the synchronizing errors in (5) and (6) converge to the zero quickly, that is, the motion synchronization of master and slave robots is achieved and stable. From Figure 5, we can see that, although the initial positions of the master and slave robots are different and both are not zero, the joint trajectories of slave robot quickly track the joint trajectories of master robot. Finally, from Figure 8, we can see that the human force $F_h$ tracks environmental for $F_e$ quickly, which means we can see $F_h$ as the environmental force $F_e$. The jumping modes are shown in Figure 9. From these figures, we can see that the designed controller is also effective.
7. CONCLUSIONS

In this paper, adaptive NN control of bilateral teleoperation system is investigated in the presence of dynamics uncertainty, unsymmetrical stochastic delays in communication channel and unknown external disturbance. By partial feedback linearization using nominal dynamics, the nonlinear dynamics of the teleoperation system is transformed into two subsystems. We propose adaptive NN control strategies based on LMIIs and adaptive updating parameters online. The stability of the closed-loop system and the boundedness of tracking errors are proved using Lyapunov stability synthesis under specific LMI conditions. The proposed controls are robust against motion disturbances and parametric uncertainties and validated by simulation studies.

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REFERENCES


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