Restudy on SVD-based watermarking scheme

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1. Introduction

In [1], the authors proposed two notes for increasing invisibility and capacity of SVD-based watermarking scheme. The first note is that modifying the coefficients in column vector will cause less visible distortion than modifying the coefficients in row vector for \( U \) component of SVD. The second one is that modifying the coefficients in row vector will cause less visible distortion than modifying the coefficients in column vector for \( V^T \) component of SVD. In this paper, we also present two notes on the basis of [1] and [2], theoretical analysis and experimental results show that the proposed two notes increase the invisibility and robustness of SVD-based watermarking scheme effectively.

2. Our proposed notes

Suppose the matrix \( A \) represents the input image. For convenience, similar with [1], we assume that \( A \) is an \( N \times N \) square matrix with rank \( r, r \leq N \). Then, the SVD of \( A \) can be represented by

\[
A = U D V^T = \begin{bmatrix}
    u_{1,1} & \cdots & u_{1,N} \\
    u_{2,1} & \cdots & u_{2,N} \\
    \vdots & \ddots & \vdots \\
    u_{N,1} & \cdots & u_{N,N}
\end{bmatrix} \begin{bmatrix}
    \sigma_1 & 0 & \cdots & 0 \\
    0 & \sigma_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & \sigma_N
\end{bmatrix} \begin{bmatrix}
    v_{1,1} & \cdots & v_{1,N} \\
    v_{2,1} & \cdots & v_{2,N} \\
    \vdots & \ddots & \vdots \\
    v_{N,1} & \cdots & v_{N,N}
\end{bmatrix}^T,
\]

where \( U \) and \( V \) are \( N \times N \) orthogonal matrices and \( D \) is an \( N \) by \( N \) singular, diagonal matrix with diagonal entries satisfying \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq \sigma_{r+1} = \cdots = \sigma_N = 0 \).

According to the proposed two notes in [1], we are likely to consider that all the column vectors of \( U \) component and \( V \) component could be adopted for robust watermarking. But, in fact, would the magnitude relationship among the coefficients in each column of \( U \) component and \( V \) component be preserved under general image processing? Table 1 shows an original \( 4 \times 4 \) image block, and Table 2 shows the distorted image block by JPEG attack (\( QF = 70 \)). Tables 3 and 4 give the \( U \) compo-
vant of SVD of the original image block and the distorted image block. From Tables 3 and 4, it is easily known that only the numerical symbols of the coefficients in the first column of $U$ component are invariable, while for other columns the numerical symbols are changed. So, we consider that only the first column of $U$ component preserves the invariable magnitude relationship.

We chose a gray scale image with size of $256 \times 256$ for experiment. Partition the image into non-overlapping blocks with size of $4 \times 4$ for each block, and perform SVD on each block. Then, the original image is distorted by JPEG attack ($QF = 70$), adding gaussian noise with variance of 0.03, and adding salt and pepper noise with density of 0.03, respectively. Finally partition these distorted images into non-overlapping blocks with the same size for each block, and perform SVD on each block. We get the number of image blocks with invariable magnitude relationship corresponding to each column of $U$ component and $V$ component. Tables 5 and 6 show the results of the experiment.

From the above two tables, it is easily known that only the first column of $U$ component and $V$ component preserves the magnitude relationship among the coefficients stably. Using other images for experiments, we can obtain the similar results. In order to verify the validity of the experimental results, here, we give our theoretical analysis.

Similarly, suppose the matrix $A$ represents the input image, and the matrix $B$ represents the distorted image. Generally, $A \geq 0$, $B = A + \Delta A \geq 0$, $\Delta A$ is the perturbation matrix, and $A, B, A^T A, B^T B$ are all irreducible matrices. Besides, $V = (V_1, V_2, V_3, \ldots)$ is obtained from eigenvectors of $A^T A$ by identity orthogonality criterion. $V_i$ represents each column of $V$, $i=1,2,3,$ and so on. The SVD of $A$ and $B$ can be represented by

$$A = U \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} V^T$$

$$B = P \begin{bmatrix} \Sigma_2 & 0 \\ 0 & 0 \end{bmatrix} Q^T,$$

for $V_1$ is an identity eigenvector corresponding to the biggest eigenvalue of $A^T A$, which is called spectral radius of $A^T A$ and represented as $\rho(A^T A)$. So we found that only the numerical symbols of the coefficients of $V_1$ are invariable, while numerical symbols of the coefficients of other columns may be changed under perturbation matrix $\Delta A$.

Moreover, from Frobenius theorem, shown as the following, we know that $\rho(A^T A)$ has positive eigenvector, suppose the eigenvector is $x$, and $x > 0$, so the coefficients of $V_1$ are all positive.

Frobenius theorem: Suppose $D \in \mathbb{C}^{n \times n}, D \geq 0$, then

$$\min_{1 \leq i \leq n} \sum_{j=1}^{n} d_{ij} \leq r(D) \leq \max_{1 \leq i \leq n} \sum_{j=1}^{n} d_{ij}$$

$$\min_{1 \leq i \leq n} \sum_{j=1}^{n} d_{ij} \leq r(D) \leq \max_{1 \leq i \leq n} \sum_{j=1}^{n} d_{ij},$$

for $U_1 = AV_1 \Sigma_1^{-1}$, $U_1$ represents the first column of $U$, so the numerical symbols of the coefficients of $U_1$ are determined by $A x$. $A$ is a nonnegative irreducible matrix, and $x > 0$, so $A x > 0$. Hence, the coefficients of $U_1$ are all positive.

On the other hand, if we adopt negative eigenvector $x$, that is, $x < 0$, then the coefficients of $V_1$ and $U_1$ will all be negative. So we get this conclusion that the coefficients of $V_1$ and $U_1$ have the same numerical symbol.

Similarly, the coefficients of the first column of $P$ and $Q$ also have the same numerical symbol. Generally, Matlab software adopts negative eigenvector $x$, so we found from our experimental results that only the numerical symbols of the coefficients of the first column of $U, V, P$ and $Q$ are invariable.

Summarize the above analysis, the first column of $U$ component and $V$ component corresponding to spectral radius of $A^T A$ is most stable, and the numerical symbols of their elements are invariable under general image processing.
From the anterior theoretical analysis and experimental results, we give our first proposed note depicted as the following:

**Note 1**: Only the coefficients in the first column of $U$ component and $V$ component may be modified for robust SVD-based watermarking.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>$U$ component of SVD of original image block</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5014</td>
<td>-0.4934</td>
</tr>
<tr>
<td>-0.5014</td>
<td>-0.4934</td>
</tr>
<tr>
<td>-0.5014</td>
<td>0.3974</td>
</tr>
<tr>
<td>-0.4959</td>
<td>0.5960</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>$U$ component of SVD of distorted image block</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5015</td>
<td>0.8515</td>
</tr>
<tr>
<td>-0.4984</td>
<td>-0.3560</td>
</tr>
<tr>
<td>-0.4984</td>
<td>-0.3560</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Number of image blocks with invariable magnitude relationship corresponding to each column of $U$ component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocks</td>
<td>Ratio</td>
</tr>
<tr>
<td>Column 1</td>
<td>4096</td>
</tr>
<tr>
<td>Column 2</td>
<td>0</td>
</tr>
<tr>
<td>Column 3</td>
<td>0</td>
</tr>
<tr>
<td>Column 4</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Number of image blocks with invariable magnitude relationship corresponding to each column of $V$ component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocks</td>
<td>Ratio</td>
</tr>
<tr>
<td>Column 1</td>
<td>4096</td>
</tr>
<tr>
<td>Column 2</td>
<td>0</td>
</tr>
<tr>
<td>Column 3</td>
<td>0</td>
</tr>
<tr>
<td>Column 4</td>
<td>0</td>
</tr>
</tbody>
</table>
For explaining the second proposed note easily, we assume that the input image $A$ is a $4 \times 4$ matrix and the SVD of $A$ is given by

$$A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ A_5 & A_6 & A_7 & A_8 \\ A_9 & A_{10} & A_{11} & A_{12} \\ A_{13} & A_{14} & A_{15} & A_{16} \end{bmatrix} = U D V^T = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \begin{bmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & b_4 \end{bmatrix} \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ c_5 & c_6 & c_7 & c_8 \\ c_9 & c_{10} & c_{11} & c_{12} \\ c_{13} & c_{14} & c_{15} & c_{16} \end{bmatrix}^T. \quad (6)$$

Perform the matrix multiplications for $UDV^T$, each pixel is given by

$$A_i = a_1 b_1 c_1 + a_2 b_2 c_2 + a_3 b_3 c_3 + a_4 b_4 c_4; \quad A_i = a_1 b_1 c_5 + a_2 b_2 c_6 + a_3 b_3 c_7 + a_4 b_4 c_8$$

$$A_i = a_1 b_1 c_9 + a_2 b_2 c_10 + a_3 b_3 c_11 + a_4 b_4 c_12; \quad A_i = a_1 b_1 c_{13} + a_2 b_2 c_{14} + a_3 b_3 c_{15} + a_4 b_4 c_{16}$$

In [1,3], the original image is first partitioned into blocks evenly and the SVD is performed on each block. The watermark is embedded by changing the relation between the second and third coefficients in the first column of $U$ component. If the embedded watermark bit is 1, the coefficients $u_{2,1}$ and $u_{3,1}$ must be modified to satisfy the condition $|u_{2,1}| - |u_{3,1}| \geq T$, where $|x|$ denotes the absolute value of $x$ and $T$ is the threshold. Otherwise, the condition $|u_{2,1}| - |u_{3,1}| \geq T$ must be held. Larger threshold $T$ results in watermarked image with lower PSNR (peak signal-to-noise ratio), but makes the embedded watermark more robust.

In order to obtain high PSNR and larger threshold $T$, which means high robustness, on the basis of [1], here we give our second proposed note for robust SVD-based watermarking.

Note 2: Use $U$ component or $V$ component to embed watermark bit, and adopt $V$ component or $U$ component to compensate visible distortion when embedding watermarking into component of SVD.

Take the input image with size $4 \times 4$ for example, $a_5$ and $a_9$ corresponding to $u_{2,1}$ and $u_{3,1}$ are modified by adding $A_1$ and $A_2$, respectively, then we can modify $c_1$, $c_5$, $c_9$, $c_{13}$ for compensating the visible distortion introduced by modifying $U$ component. Assume that the modified quantity is $A_3$, $A_4$, $A_5$, and $A_6$, respectively. Define the modified $A_5$, $A_6$, $A_7$, $A_8$, $A_9$, $A_{10}$, $A_{11}$, $A_{12}$ as $A_5$, $A_6$, $A_7$, $A_8$, $A_9$, $A_{10}$, $A_{11}$, $A_{12}$ when compensatory operation is not used, and the modified $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$, $A_7$, $A_8$, $A_9$, $A_{10}$, $A_{11}$, $A_{12}$ when compensatory operation is adopted. From the equation of PSNR, as long as $A_3$, $A_4$, $A_5$, and $A_6$ satisfy the following condition, then the lower visible distortion can be obtained

$$\sum_{i=1}^{16} (A''_i - A_i)^2 < \sum_{i=5}^{12} (A'_{i} - A_{i})^2, \quad (8)$$

when $A_1 = e_1 = \frac{c_1 a_1}{a_5 + a_9}$, then $A_5$ will have no change. When $A_3 = e_2 = \frac{c_1 a_1}{a_5 + a_9}$, then $A_9$ will have no change. Hence, in the following experiments, we adopt the average value of $e_1$ and $e_2$ to endow with $A_3$. Similarly, $A_4$, $A_5$, and $A_6$ have adopted these values shown as the following:

$$A_3 = -\frac{1}{2} \left( \frac{c_1 a_1}{a_5 + a_1} + \frac{c_1 a_2}{a_9 + a_2} \right) \quad (9)$$

$$A_4 = -\frac{1}{2} \left( \frac{c_5 a_1}{a_5 + a_1} + \frac{c_5 a_2}{a_9 + a_2} \right) \quad (10)$$

$$A_5 = -\frac{1}{2} \left( \frac{c_9 a_1}{a_5 + a_1} + \frac{c_9 a_2}{a_9 + a_2} \right) \quad (11)$$

$$A_6 = -\frac{1}{2} \left( \frac{c_{13} a_1}{a_5 + a_1} + \frac{c_{13} a_2}{a_9 + a_2} \right), \quad (12)$$

if they do not satisfy (8), then no modification is performed on $c_1$, $c_5$, $c_9$, and $c_{13}$; otherwise modify $c_1$, $c_5$, $c_9$, and $c_{13}$ for lower visible distortion. The lower visible distortion, the larger threshold $T$ can be chosen, which means higher robustness.

3. Experimental results

In this section, several experiments are performed to verify the effectiveness when incorporating our proposed two notes for SVD-based watermarking scheme. Two gray images with size $512 \times 512$, “Lena” and “Peppers”, are used as original images. And a binary image with size $32 \times 32$ is used as robust watermark.
Table 7 shows the PSNR performance under different thresholds. The anterior two rows show the PSNR values of embedding watermark into $U$ component only by using method in [1]. The latter two rows give the PSNR values of using the proposed two notes. From the comparison, it is easily known that our proposed notes can improve the PSNR values greatly. When PSNR value is enhanced, then larger threshold value may be adopted, which means higher robustness. Table 8 shows the ratios of compensated block to total used block under different thresholds. It illuminates that our compensatory operation is useful for reducing visible distortion. For the case of $T = 0.012$, we apply different attacks to the watermarked image to assess robustness. The attacks include JPEG compression with the compressed file being approximately 38% to original file size, an addition of salt and pepper noise with density of 0.03, contrast enhancement 50%, supplement image attack. The error rate between the extracted watermark and the original watermark is used to assess the robustness of embedded watermark. The error rates of the above four attacks are shown in Table 9. The error rate is lower, the robustness is higher. From Table 9, we can know that our proposed notes for SVD-based watermarking scheme own high robustness against common image processing operations.

4. Conclusions

In this paper, we proposed two notes for SVD-based robust watermarking scheme, experimental results and theoretical analysis both show that the two notes increase the invisibility and robustness of SVD-based watermarking scheme. Future work will focus on using optimization algorithm to search the best values for $A_1$, $A_4$, $A_5$, and $A_6$ and obtain the least visible distortion.

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References
