Adaptive Fuzzy Finite-Time Coordination Control for Networked Nonlinear Bilateral Teleoperation System

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Abstract—The master–slave control design problem is considered for the networked teleoperation system with friction and external disturbances. A new finite-time synchronization coordination control method is proposed with the help of adaptive fuzzy approximation. We develop a new nonsingular fast terminal sliding mode (NFTSM) to provide faster convergence and higher precision than the linear hyperplane-sliding mode and the classic terminal-sliding mode (TSM). Then, the adaptive fuzzy logic system is employed to approximate the system uncertainties, and the corresponding adaptive fuzzy NFTSM controller is designed. By constructing Lyapunov function, the stability and finite-time synchronization performance are proved with the new controller in the presence of system uncertainties and external disturbances. Compared with the traditional teleoperation design method, the new control scheme achieves better transient-state performance and steady-state performance. Finally, the simulations are performed and the comparisons are shown among the proposed method, the P+d method, the PD+d method, the DFF method, and the classic TSM FTSM. The simulation results further demonstrate the effectiveness of the proposed method.

Index Terms—Adaptive fuzzy control, finite-time control, teleoperation system, time delay.

I. INTRODUCTION

The teleoperation system was first conceived to allow a human operator to perform dangerous tasks on a remote site, while remaining in a safe place. A typical teleoperation is composed of five parts: human operator, master robot, communication channel, slave robot, and external environment [1]. Compared with the full-automatic system, the human operator makes the system more flexible. Today, applications of master–slave telerobotic systems can be found in many areas from micro- to macro-scales, for example, space operation, underwater exploration, handling of hazardous materials, and telesurgery (see, [1], and the references cited therein).

Many methods have been proposed to deal with the teleoperation system design problem. The passivity-based method was developed to overcome the instability of the teleoperation system caused by constant time delay [2]. Later, Niemeyer and Slotine [3] introduced the wave variables following the former scattering transformation. Subsequent schemes, which have been built on the two aforementioned approaches, have been suggested in [4] and [5]. After that, the passivity-based architecture was extended in [6] to guarantee state synchronization of master/slave without using scattering transformation. Following this line, in [7], a global stable P+d controller was proposed, and the stability conditions were developed for the constant time delay case without the delayed derivative action. In addition, Hua and Liu [8] proposed the delay dependent linear matrix inequality stability criteria for the closed-loop teleoperation system with time-varying delays. In [9], a kinematic control framework for asymmetric teleoperation systems was proposed. Considering the case that the velocity signals cannot be obtained directly, Hua and Liu [10] proposed a velocity observer-based new controller. Moreover, the exponential synchronization performance was achieved. However, in the aforementioned works, the asymptotic/exponential synchronization performances were achieved. This means that the synchronization error converges to zero when time goes to infinity. It is well known that for an ideal teleoperation system, we expect the slave can follow the master quickly, and the synchronization performance can be achieved in finite time.

The finite-time control problem will be considered in this paper by using the sliding mode control (SMC) method. SMC has been widely used because of its relative simplicity of implementation and robustness against both plant uncertainties and external disturbances. However, the classic linear hyperplane-based sliding mode can only guarantee asymptotic error convergence [11]–[13]. Thus, the finite-time control theory appeared to get a higher convergence speed. Terminal-sliding mode control (TSMC) is an effective finite-time control approach, and it was first proposed in [14]. Similar to the linear sliding mode (LSM) technique, strong robustness with respect to uncertain dynamics can be obtained. Moreover, the tracking error converges to zero in finite time. However, the premier papers about TSMC encountered the singularity problem. To deal with the singularity problem, a nonsingular terminal sliding manifold...
was proposed in [15]. Then, a globally continuous nonsingular TSMC method was presented in [16]. Afterward, fuzzy logic system and a continuous nonsingular TSM were combined to control a manipulator system in [17]. Different from the direct method, an indirect method was proposed in [18] to avoid the singularity problem, and the modified terminal sliding manifold could produce a smoother switch. Motivated by [18], a Chebyshev neural network-based nonsingular TSM controller was presented in [19] for spacecraft control. In [20], some new forms of fast TSM strategies were presented, and comparisons of the convergence speed were made between the different fast TSMC.

It is well known that the robotic manipulator is a complicated, dynamically coupled, highly time varying, and highly nonlinear system. Moreover, with the limitation of modeling method and the complex external task environment, the teleoperation system is inevitably subject to structured and unstructured uncertainties. A fuzzy logical system (FLs) was introduced by Zadeh in 1965, as one of the most popular intelligent computation approaches, which was proved to be an essential tool for solving some various classes of engineering problems. To approximate the system uncertainties and external disturbance, the FLs was widely used in robotic control [21], [22]. Recently, the adaptive fuzzy approximation approach was also used in controlling MIMO nonlinear systems and nonlinear time-delay systems [23]–[30]. Tong et al. [31] proposed a fuzzy output feedback controller that combines the backstepping method and the stochastic small-gain approach. The adaptive sliding-mode controllers for the Takagi–Sugeno (T–S) fuzzy system with mismatched uncertainties and exogenous disturbances were proposed in [32]. A way of estimating learning errors for direct adaptive fuzzy control systems was proposed in [33]. The adaptive fuzzy method was used in controlling the teleoperation system under time-varying delay in [34] and [35]. In this paper, we will use the adaptive fuzzy method to deal with the uncertainties in the master–slave model.

We propose a new teleoperation design scheme for the networked teleoperation system with uncertainties. A novel TSMC method is presented first. Then, by employing the adaptive fuzzy approximation method, we design master–slave controllers. With the new controllers, the finite-time synchronization performance is achieved. The main contributions of this paper are stated as follows: 1) A new FTSM is proposed. Compared with TSM control method, the faster transient-state and higher-precision control performances are both achieved. 2) The FLs and parameter adaptive method are combined to deal with the system uncertainties and external disturbance, and the system stability is rigorously proved with the designed adaptive fuzzy controllers. 3) The precise reaching time and the sliding time can be computed with system initial states, sliding mode, and controller parameters.

This paper is organized as follows. Section II presents some preliminary knowledge for the dynamics of teleoperation system with the related properties and the knowledge of FLs. In Section III, the new controllers for the teleoperator are presented. The simulation results are presented in Section IV. Finally, Section V concludes with a summary of the obtained results.

II. SYSTEM FORMULATION

A. Dynamics of a Teleoperator

Consider a master–slave system given by the following model:

\[
\begin{align*}
M(q_m) \ddot{q}_m + C_m(q_m, \dot{q}_m) \dot{q}_m + F_m \dot{q}_m + f_{cm}(\dot{q}_m) + \tau_{dm} + g_m(q_m) &= \tau_m + F_h, \\
M_s(q_s) \ddot{q}_s + C_s(q_s, \dot{q}_s) \dot{q}_s + F_s \dot{q}_s + f_{cs}(\dot{q}_s) + \tau_{ds} + g_s(q_s) &= \tau_s + F_e,
\end{align*}
\]

where \(q_m(t), q_s(t) \in \mathbb{R}^{n \times 1}\) are the vectors of the joint displacements; \(\dot{q}_m(t), \dot{q}_s(t) \in \mathbb{R}^{n \times 1}\) are the vectors of joint velocities; \(\ddot{q}_m(t), \ddot{q}_s(t) \in \mathbb{R}^{n \times 1}\) are the vectors of joint accelerations; \(M_m(q_m), M_s(q_s) \in \mathbb{R}^{n \times n}\) are the positive definite inertia matrices; \(C_m(q_m, \dot{q}_m), C_s(q_s, \dot{q}_s) \in \mathbb{R}^{n \times n}\) are the matrices of centripetal and coriolis torques; \(F_m, F_s\) denote the viscous friction coefficients; \(f_{cm}(\dot{q}_m), f_{cs}(\dot{q}_s) \in \mathbb{R}^{n \times 1}\) are the Coulomb friction coefficients; \(\tau_{dm}, \tau_{ds} \in \mathbb{R}^{n \times 1}\) denote the bounded unknown external disturbances; \(g_m(q_m), g_s(q_s) \in \mathbb{R}^{n \times 1}\) are the gravitational torques; \(F_h, F_e \in \mathbb{R}^{n \times 1}\) are the torques applied by the human operator and the remote environment, respectively; and \(\tau_m, \tau_s \in \mathbb{R}^{n \times 1}\) are the applied torques. The well-known properties for a robot system are revisited here.

**Property 1:** The inertia matrix \(M_i(q_i)\) is a symmetric positive definite function and there exist positive constants \(m_{i_1}\) and \(m_{i_2}\) such that \(m_{i_1} I \leq M_i(q_i) \leq m_{i_2} I\), with \(i = m, s\).

**Property 2:** The matrix \(M_i(q_i) - 2C_i(q_i, \dot{q}_i)\) is skew symmetric.

**Property 3:** For all \(q_i, x, y \in \mathbb{R}^{n \times 1}\), there exists a positive scalar \(a_i\) such that

\[
\|C_i(q_i, x)y\| \leq a_i \|x\| \|y\|
\]

where \(\|\cdot\|\) denotes the Euclidean norm of a vector and the corresponding induced matrix norm.

**Property 4:** The gravity vector \(g_i(q_i)\) is bounded as it consists of sinusoidal function \(g_i\). There exists a positive scalar \(\mu_{gi}\) such that \(\|g_i(q_i)\| \leq \mu_{gi}\).

**Remark 1:** In this paper, both the viscous friction and the Coulomb friction are considered in the master robot and the slave robot, respectively. Here, the Coulomb friction function \(f_{ci}(\dot{q}_i)\) is a bounded and piecewise continuous function.

B. Fuzzy Logic Systems

During the past years, FLs have been extensively used as universal approximators for the design of dynamic systems with precise model unknown. A fuzzy system is a collection of fuzzy IF–THEN rules of the form

\[
R^{(j)}: \text{IF} \ z_1(t) \text{ is } A_{1j}^{(j)} \text{ and } \cdots \text{ and } z_n(t) \text{ is } A_{nj}^{(j)} \text{ THEN } y(t) \text{ is } B_{j}^{(j)},
\]
By using the strategy of singleton fuzzification, product inference, and center-average defuzzification, the output of the fuzzy system is

\[ y(z(t)) = \sum_{j=1}^{q(t)} \mu_{A_j}^i(z(t)) \prod_{j=1}^{q(t)} \mu_{A_j}^i(z(t)) \]

(2)

where \( \mu_{A_j}^i(z(t)) \) is the membership function of linguistic variable \( z_i(t) \), and \( y^j \) is the point in \( \mathbb{R} \) at which \( \mu_{B_j} \) achieves its maximum value (assume that \( \mu_{B_j}(y^j) = 1 \)).

Introducing the concept of the fuzzy basic vector function \( \varsigma(z(t)) \), gives

\[ y(z(t)) = \theta(t)^T \varsigma(z(t)) \]

(3)

where

\[ \theta(t) = (y^1(t), y^2(t), \ldots, y^q(t))^T \]

\[ \varsigma(z(t)) = (\varsigma_1(z(t)), \varsigma_2(z(t)), \ldots, \varsigma_{q(t)}(z(t)))^T \]

and \( \varsigma_j(z(t)) \) is defined as

\[ \varsigma_j(z(t)) = \prod_{i=1}^{q(t)} \mu_{A_j}^i(z_i(t)) \prod_{i=1}^{q(t)} \mu_{A_j}^i(z_i(t)) \]

Based on the universal approximation theorem, there exists the optimal approximation parameter \( \theta^* \) such that \( \theta^T \varsigma(z(t)) \) can approximate a nonlinear function \( G(z(t)) \) to any desired degree over a compact set \( \Omega_1 \). The parameter \( \theta^* \) is defined as follows:

\[ \theta^* = \arg \min_{\theta \in \Omega_1} \left( \sup_{z(t) \in \Omega_2} \left| \theta^T (z(t)) - G(z(t)) \right| \right) \]

(4)

where \( \Omega_1 \) and \( \Omega_2 \) denote the sets of suitable bounds on \( \theta(t) \) and \( z(t) \), respectively. The minimum approximation error satisfies

\[ G(z(t)) = \theta^T \varsigma(z(t)) + \epsilon(z(t)) \]

(5)

\[ \epsilon(z(t)) = \epsilon(z(t)) + G_2(z(t)) \]

where \( \epsilon(z(t)) \) is the piecewise function approximation error, and \( \epsilon^* \) is an upper bound of the approximation error.

**Definition 1:**

\[ \text{sign}(\xi)^n = [\xi_1^n \text{sign}(\xi_1), \xi_2^n \text{sign}(\xi_2), \ldots, \xi_n^n \text{sign}(\xi_n)]^T \]

(6)

where \( \xi = [\xi_1, \xi_2, \ldots, \xi_n]^T \in \mathbb{R}^n \), \( \alpha_1, \alpha_2, \ldots, \alpha_n > 0 \) and \( \text{sign}(\cdot) \) being the standard signum function.

**Lemma 1 (sliding time) [20]:** Choose the terminal-sliding mode as follows:

\[ s = \dot{e} + \alpha \text{sign}(e)^\gamma_1 + \beta_{12} \text{sign}(e)^\gamma_2 \]

(7)

where \( \alpha > 0, \beta > 0, \gamma_1 \geq 1 \) and \( 0 < \gamma_2 < 1 \). If \( s = 0 \), the convergence time \( T_1 \) of \( e \) is about

\[ T_1 = \int_0^{e_{i0}} \frac{1}{\alpha e^\gamma_1 + \beta e^\gamma_2} \text{d}e = \frac{|e_{i0}|^{1-\gamma_1}}{1-\gamma_1} \alpha/(1-\gamma_1) \alpha(1-\gamma_1)/\gamma_1 \]

(8)

where \( e_{i0} = e_i(0) \). \( F() \) denotes Gauss’ Hypergeometric function [36], and the exact form of \( F() \) changes with the involved parameters. For example

\[ F(1, 1; 2; Z) = -Z^{-1} \ln(1 - Z); \]

\[ F\left(\frac{1}{2}, 1; \frac{3}{2}; -Z^2\right) \]

\[ = Z^{-1} \arctan(Z) \]

\[ i \in \{1, 2, \ldots, n\} \]

**Lemma 2 (reaching time) [16]:** Consider the dynamics model

\[ \dot{x} = f(x), \quad f(0) = 0 \quad \text{and} \quad x \in \mathbb{R}^n \].

If there is a positive definite scalar function \( V(x) \) such that

\[ \dot{V}(x) \leq -\alpha V(x) - \beta V(x)^\delta \]

(9)

where \( \alpha, \beta > 0, \quad 0 < \delta < 1 \), then the system is finite-time stable. Furthermore, the settling time is given by

\[ T \leq \frac{1}{\alpha(1-\delta)} \ln \frac{\alpha V(1-x_0) + \beta}{\beta} \]

(10)

**Lemma 3 [16]:** Assume \( a_1 > 0, a_2 > 0, \) and \( 0 < c < 1 \), the following inequality holds:

\[ (a_1 + a_2)^c \leq a_1^c + a_2^c \]

(11)

The NFTSM that is proposed in this paper is given as follows:

\[ s_m = \dot{e}_m + \alpha_{m1} \text{sign}(e_m)^{\gamma_{m1}} + \alpha_{m2} \beta_m (e_m) = \dot{e}_m + \lambda_m (e_m) \]

\[ s_n = \dot{e}_s + \alpha_{s1} \text{sign}(e_s)^{\gamma_{s1}} + \alpha_{s2} \beta_s (e_s) = \dot{e}_s + \lambda_s (e_s) \]

(12)

where \( \alpha_{m1}, \alpha_{m2}, \text{and} \alpha_{s2} \) are positive constants, \( \gamma_{m1} > 1, \gamma_{s1} > 1; e_m = q_m(t - T_m) - q_m; \) and \( e_s = q_m(t - T_m) - q_s \) are the position synchronization errors between the master manipulator and the slave manipulator; \( \dot{e}_m = \dot{q}_m(t - T_m) - \dot{q}_m; \) and \( \dot{e}_s = \dot{q}_m(t - T_m) - \dot{q}_s \) are the velocity-tracking errors; \( T_m \) represents the signal transmission time delay from the master side to the slave side, and \( T_s \) stands for the transmission time delay from the slave side to the master side. The time delay \( T_m \) and \( T_s \) are not known in advance. The \( \beta_m (e_m) \) and \( \beta_s (e_s) \) are
defined as follows:

\[
\beta_m(e_m) = \begin{cases} 
\text{sign}(e_m)^{\gamma_m} & \text{if } \bar{s}_m = 0 \text{ or } \bar{s}_m \neq 0, \quad |e_m| > \mu_m \\
\kappa_m e_m + \kappa_m^2 \text{sign}(e_m) e_m^2 & \text{if } \bar{s}_m \neq 0, \quad |e_m| \leq \mu_m
\end{cases}
\]

and

\[
\beta_s(e_s) = \begin{cases} 
\text{sign}(e_s)^{\gamma_s} & \text{if } \bar{s}_s = 0 \text{ or } \bar{s}_s \neq 0, \quad |e_s| > \mu_s \\
\kappa_s e_s + \kappa_s^2 \text{sign}(e_s) e_s^2 & \text{if } \bar{s}_s \neq 0, \quad |e_s| \leq \mu_s
\end{cases}
\]

where \( \gamma_m < 1 \), \( \gamma_s < 1 \), \( \bar{s}_m = \bar{e}_m + \alpha_m e_m, \quad \text{sign}(e_m)^{\gamma_m} = \bar{e}_m + \alpha_m \text{sign}(e_m)^{\gamma_m + 1} \), \( \bar{s}_s = \bar{e}_s + \alpha_s \text{sign}(e_s)^{\gamma_s + 1} \), \( \kappa_m = (2 - \gamma_m)\mu_m^{\gamma_m - 1} \), \( \kappa_s = (\gamma_s - 1)\mu_s^{\gamma_s - 2} \), \( \mu_m, \mu_s \) are small positive constants.

Remark 2: Compared with the LSM, the system with the TSM can get faster convergence and higher control precision. However, the classic TSM faces the singularity problem. To deal with the singularity problem, Wang et al. [18] proposed a new TSM by using the switching idea. However, the convergence speed in [18] was in a relatively slow rate when the system states are far away from the equilibrium points. Considering the speed problem, the comparisons between the different TSM were made in [20]. Motivated by [18] and [20], we proposed a modified NFTSM. The new NFTSM not only can avoid the singularity problem, but can also get a higher convergence speed.

Remark 3: The new NFTSM is composed of three parts. When the system states stay at a distance from equilibrium points, \( \alpha \text{sign}(e)^{\gamma} \) dominates over \( \beta \text{sign}(e)^{\gamma} \), thus a fast convergence rate can be guaranteed; when the system states are close to the region, the dominant term \( \beta \text{sign}(e)^{\gamma} \) determines finite-time convergence.

B. Controller Design

With the NFTSM, the teleoperation dynamics can be rewritten as the following forms:

\[
\begin{align*}
M_m(q_m) \dot{s}_m + C_m(q_m, \dot{q}_m) s_m &= G_m'(Z_m) - \tau_m - F_h + \tau_{dm} \\
M_s(q_s) \dot{s}_s + C_s(q_s, \dot{q}_s) s_s &= G_s'(Z_s) - \tau_s - F_s + \tau_{ds}
\end{align*}
\]

where \( Z_m = [\ddot{q}_m'(t - T_s), \dot{q}_m'(t - T_s), q_m'(t - T_s), q_m(t - T_s), q_m'(t - T_s)]^T \), \( Z_s = [\dot{q}_s'(t - T_m), \dot{q}_s'(t - T_m), q_s'(t - T_m), q_s'(t - T_m), q_s'(t - T_m)]^T \), \( G_m'(Z_m) \) and \( G_s'(Z_s) \) are defined as

\[
G_m'(Z_m) = M_m(q_m) \dot{q}_m(t - T_m) + \lambda_m e_m)
+C_m(q_m, \dot{q}_m)(\dot{q}_m(t - T_m) + \lambda_m e_m)
+F_m q_m + f_{cm}(q_m) + g_m(q_m)
\]

and

\[
G_s'(Z_s) = M_s(q_s)(\dot{q}_s'(t - T_m) + \lambda_s e_s)
+C_s(q_s, \dot{q}_s)(\dot{q}_s'(t - T_m) + \lambda_s e_s)
\times (\dot{q}_m'(t - T_m) + \lambda_m e_s) + F_s q_s + f_{cs}(\dot{q}_s) + g_s(q_s)
\]

Based on the FLs approximation property, we use the functions \( \hat{G}_m'(Z_m) \) and \( \hat{G}_s'(Z_s) \) to approximate the functions \( G_m'(Z_m) \) and \( G_s'(Z_s) \) with

\[
\hat{G}_m'(Z_m) = \hat{\theta}_m^T s_m(Z_m), \quad \hat{G}_s'(Z_s) = \hat{\theta}_s^T s_s(Z_s)
\]

where \( i = m, s; \hat{\theta}_i \) is a matrix of the fuzzy adaption parameters; \( \varsigma_i(Z_i) \) is a vector that denotes a known fuzzy basis function.

Additionally, to streamline the presentation, we give the following definitions:

\[
\tilde{G}_m'(Z_m) = G_m'(Z_m) - \hat{G}_m'(Z_m)
= (\theta_m^T - \hat{\theta}_m^T) s_m(Z_m) + \bar{e}_m(Z_m)
= \tilde{\theta}_m^T s_m(Z_m) + \bar{e}_m(Z_m)
\]

and

\[
\tilde{G}_s'(Z_s) = G_s'(Z_s) - \hat{G}_s'(Z_s)
= (\theta_s^T - \hat{\theta}_s^T) s_s(Z_s) + \bar{e}_s(Z_s)
= \tilde{\theta}_s^T s_s(Z_s) + \bar{e}_s(Z_s).
\]

Because of the existence of piecewise continuous functions \( f_{cm}(\dot{q}_m) \) and \( f_{cs}(\dot{q}_s) \), \( \bar{e}_i(Z_i) = \epsilon_i(Z_i) + f_{ci}(\dot{q}_i) \), where \( i = m, s \).

The adaptive fuzzy control scheme is shown in Fig. 1 and the controllers are as follows:

\[
\tau_m = \tilde{G}_m'(Z_m) + K_{m1} s_m + K_{m2} \text{sign}(s_m)^{\rho} + \bar{w}_m \text{sign}(s_m) + \xi_m \text{sign}(s_m)
\]

\[
\tau_s = \tilde{G}_s'(Z_s) + K_{s1} s_s + K_{s2} \text{sign}(s_s)^{\rho} + \bar{w}_s \text{sign}(s_s) + \xi_s \text{sign}(s_s)
\]

where \( \bar{w}_m \) and \( \bar{w}_s \) are used to estimate the upper bound of the sum of fuzzy modeling error, and the bounded disturbances \( w_m \) and \( w_s \), i.e., \( \| \bar{e}_m(Z_m) + \tau_{dm} \| \leq w_m \) and \( \| \bar{e}_s(Z_s) + \tau_{ds} \| \leq w_s \). In addition, \( K_{m1}, K_{m2}, K_{s1}, \) and \( K_{s2} \) are positive diagonal matrices; \( \xi_m \) and \( \xi_s \) are positive scalars, \( \xi_m \) and \( \xi_s \) will be defined; \( 0 < \rho < 1 \).
Remark 4: For practical teleoperation design, synchronization accuracy and synchronization time are very important. For example, teleurgery can minimize health care cost and make specialist doctors available throughout the world saving people’s lives and improving health care systems. Meanwhile, teleurgery requires a higher performance than other applications because the work performance influences the patient’s health status and even his life directly. However, the existing teleoperation control design methods can guarantee that the synchronization error converges to zero asymptotically. It is well known that the state variables reach zero when $t \to \infty$ for the asymptotical stability. Therefore, the synchronization performance cannot suit the demand for teleurgery. The proposed control scheme (21) can guarantee the synchronization performance in the finite-time $T$ shown in (29), which can be tuned by choosing the design parameters, and then the high-precision and fast-synchronization performance are both realized.

C. Performance Analysis

With the controllers (21) for system (1), we have the following two theorems. In Theorem 1, the stability of the closed-loop teleoperation system is achieved. And the master–slave finite-time synchronization performance is obtained in the Theorem 2.

Theorem 1: Consider the teleoperator system (1) with the new controller (21) in free motion, and the FLs adaptive laws are given as

$$\dot{\theta}_m = \Lambda_m s_m (Z_m) s_m^T, \quad \dot{\theta}_s = \Lambda_s s_s (Z_s) s_s^T$$

where $\Lambda_m$ and $\Lambda_s$ are positive definite matrices, the adaptive tuning laws are

$$\dot{w}_m = \| s_m \|, \quad \dot{w}_s = \| s_s \|$$

where $\dot{w}_m (0) \geq 0$, $\dot{w}_s (0) \geq 0$, then the closed-loop teleoperation system is stable.

Proof:

Let us propose the following Lyapunov function candidate

$$V = \frac{1}{2} s_m^T M_m (q_m) s_m + \frac{1}{2} s_s^T M_s (q_s) s_s + \frac{1}{2} \text{trace} (\hat{\theta}_m^T \Lambda_m^{-1} \hat{\theta}_m) + \frac{1}{2} \text{trace} (\hat{\theta}_s^T \Lambda_s^{-1} \hat{\theta}_s) + \frac{1}{2} (\hat{w}_m - w_m)^2 + \frac{1}{2} (\hat{w}_s - w_s)^2$$

(24)

evaluating $V$ along the system trajectories, yields

$$\dot{V} = s_m^T M_m (q_m) \dot{s}_m + \frac{1}{2} s_m^T M_m (q_m) s_m + s_s^T M_s (q_s) s_s$$

$$+ \frac{1}{2} \text{trace} (\hat{\theta}_m^T \Lambda_m^{-1} \hat{\theta}_m) - \text{trace} (\hat{\theta}_m^T \Lambda_m^{-1} \hat{\theta}_m)$$

$$+ (\hat{w}_m - w_m) \dot{w}_m + (\hat{w}_s - w_s) \dot{w}_s$$

(25)

using the property 2 of the robot manipulators with the controller (21) and the FLs adaptive laws (22), yields

$$\dot{V} = s_m^T (G_m^r (Z_m) - \dot{G}_m^r (Z_m) - K_m s_m - K_m s_m \text{sig}(s_m)^p)$$

$$- \hat{w}_m \| s_m \| - \xi_m \| s_m \| + \tau_{dm}) + s_s^T (G_s^r (Z_s) - \dot{G}_s^r (Z_s)$$

$$- K_s s_s - K_s \text{sig}(s_s)^p - \hat{w}_s \| s_s \| - \xi_s \| s_s \| + \tau_{ds})$$

$$- \text{trace} (\hat{\theta}_m^T \Theta_m (Z_m) s_m^T s_m) - \text{trace} (\hat{\theta}_s^T \Theta_s (Z_s) s_s^T s_s)$$

$$+ (\hat{w}_m - w_m) \dot{w}_m + (\hat{w}_s - w_s) \dot{w}_s$$

(26)

then with (19), (20), and the adaptive tuning laws (23), we can obtain

$$\dot{V} = -s_m^T K_{m1} s_m - s_m^T K_{m2} \text{sig}(s_m)^p - s_s^T K_{s1} s_s$$

$$- s_s^T K_{s2} \text{sig}(s_s)^p + s_m^T (\dot{e}_m (Z_m) + \tau_{dm}) + s_s^T (\dot{e}_s (Z_s) + \tau_{ds})$$

$$- \hat{w}_m \| s_m \| - \hat{w}_s \| s_s \| + (\hat{w}_m - w_m) \| s_m \|$$

$$+ (\hat{w}_s - w_s) \| s_s \| - \xi_m \| s_m \| - \xi_s \| s_s \|$$. (27)

Substituting the upper bound $w_m$ of $\| \epsilon_m^* + \tau_{dm} \|$, and the upper bound $w_s$ of $\| \epsilon_s^* + \tau_{ds} \|$ into (27), yields

$$\dot{V} \leq -s_m^T K_{m1} s_m - s_s^T K_{s2} \text{sig}(s_s)^p$$

$$- s_s^T K_{s1} s_s - s_s^T K_{s2} \text{sig}(s_s)^p$$

(28)

With the equations (24) and (28), the stability of the closed-loop teleoperation system can be guaranteed. Moreover, the boundedness of the $s_m$, $s_s$, $\theta_m$, $\theta_s$, $\hat{w}_m - w_m$, and $\hat{w}_s - w_s$ also can be achieved. The proof is completed.

Next, the finite-time convergence performance will be proved.

Theorem 2: Consider the teleoperator system (1) with the new controller (21) in free motion, with the estimation errors $\hat{\theta}_m^T \Theta_m (Z_m)$ and $\hat{\theta}_s^T \Theta_s (Z_s)$, and the adaptive approximate error $w_m - \hat{\theta}_m$ and $w_s - \hat{\theta}_s$ if the design parameter $\xi_m$ is set as $\xi_m \geq || \hat{\theta}_m^T \Theta_m (X_m) || + || w_m - \hat{\theta}_m ||$ and $\xi_s$ is set as $\xi_s \geq || \hat{\theta}_s^T \Theta_s (X_s) || + || w_s - \hat{\theta}_s ||$, then the system trajectory will converge to $s_m = 0$ and $s_s = 0$ in finite time. Furthermore, the synchronization performance between the master and the slave will be obtained in finite time. And the exact convergence time is

$$T = T_1 + T_2$$

(29)

$$T_1 = \frac{1}{\alpha (1 - \delta)} \ln \frac{\alpha U^{-\delta} (x_0) + \beta}{\beta}$$

(30)

where $\alpha = \min (\lambda_{\min} (K_{m1})^{-2/m_1^2}, \lambda_{\min} (K_{s1})^{-2/m_2^2})$; $\beta = \min (\lambda_{\min} (K_{m2})^{-2/m_1^2} (1 + \rho)^2, \lambda_{\min} (K_{s2})^{-2/m_2^2} (1 + \rho)^2)$; and $\delta = (1 + \rho)/2$; $\lambda_{\min} (K_{ij})$ denotes the minimum eigenvalue of matrix $K_{ij}$.

$$T_2 = \max \left\{ \frac{| e_m (T_1) |^{1 - \gamma_m} - \gamma_m}{1 - \gamma_m} \times F \left( \frac{e_m (T_1)}{1 - \gamma_m} \right) \right\} \times \alpha_{m1} (1 - \gamma_m) \times 1$$

$$+ \frac{2 \gamma_m - 1}{1 - \gamma_m} \times -\alpha_{m1} \alpha_{m1}^{-1} | e_m (T_1) |^{2 - \alpha_m},$$

$$\frac{| e_s (T_1) |^{1 - \gamma_s} - \gamma_s}{1 - \gamma_s} \times F \left( \frac{e_s (T_1)}{1 - \gamma_s} \right)$$

$$+ \frac{2 \gamma_s - 1}{1 - \gamma_s} \times -\alpha_{s1} \alpha_{s1}^{-1} | e_s (T_1) |^{2 - \alpha_s}$$

(31)

Proof:
Select another Lyapunov function candidate
\[ U = \frac{1}{2} s^T m s + \frac{1}{2} s^T M s. \] (32)

Differentiating the Lyapunov function \( U \) along the system trajectories, yields
\[ \dot{U} = s^T m \dot{s} + \frac{1}{2} s^T M \dot{s} + s^T M s \dot{s} + \frac{1}{2} s^T M s. \] (33)
with the property 2 of the robot manipulators with the controller (21), we have
\[ \dot{U} \leq -s^T K m s - s^T M s \sin(\theta) s - s^T K a s - s^T \dot{\theta}^T s (Z m) + s^T \dot{\theta}^T s (Z s) + || s_m || (w_m - \dot{w}_m) + || s_s || (w_s - \dot{w}_s) \]
\[ - \xi_m || s_m || - \xi_s || s_s ||. \] (34)
Then, with \[ || s^T \dot{\theta}^T s (Z m) || \leq || s_m || || \dot{\theta}^T s (Z m) || \]
and \[ || s^T \theta^T s (Z s) || \leq || s_s || || \theta^T s (Z s) || \]
we can obtain
\[ \dot{U} \leq -s^T K m s - s^T M s \sin(\theta) s - s^T K a s - s^T \dot{\theta}^T s (Z m) + s^T \dot{\theta}^T s (Z s) + || s_m || (w_m - \dot{w}_m) + || s_s || (w_s - \dot{w}_s) - \xi_m || s_m || - \xi_s || s_s ||. \] (35)

With the definitions of \( \xi_m \) and \( \xi_s \), we have
\[ \dot{U} \leq -s^T K m s - s^T M s \sin(\theta) s - s^T K a s - s^T \dot{\theta}^T s (Z m) + s^T \dot{\theta}^T s (Z s) + || s_m || (w_m - \dot{w}_m) + || s_s || (w_s - \dot{w}_s) - \xi_m || s_m || - \xi_s || s_s ||. \] (36)

With the manipulator property 1, we have
\[ \dot{U} \leq - \frac{\lambda_{m1}}{m m_1} \left( \frac{2}{m_{m1}} \right) \left( \frac{1}{2} s^T m s \right) - \frac{\lambda_{m2}}{m m_2} \left( \frac{2}{m_{m2}} \right) \left( \frac{1}{2} s^T M s \right) + \frac{\lambda_{a1}}{m s_1} \left( \frac{1}{2} s^T M s \right) + \frac{\lambda_{a2}}{m s_2} \left( \frac{1}{2} s^T M s \right). \] (37)

With the Lemma 3, we have
\[ \dot{U} \leq - \min \left\{ \frac{\lambda_{m1}}{m m_1} \left( \frac{2}{m_{m1}} \right), \frac{\lambda_{a1}}{m s_1} \left( \frac{1}{2} s^T M s \right) \right\} \left. U \right|_{\theta}^+ \]
\[ = - \alpha U - \beta U \hat{\delta}. \] (38)

By using Lemma 2, the reaching time can be computed accurately as shown in (30). Moreover, with Lemma 1, the accurate sliding time can also be computed as shown in (31). This completes the proof.

**Remark 5:** Because switching idea is used to avoid the singularity problem, when the terminal-sliding mode \( s_m \) and \( s_s \) reach zero, two possible cases for synchronization errors \( e_m \) and \( e_s \), approaching zero will occur. Take the \( s_m \) as an example, at the first case, \( |e_m| \geq \mu_m \) as the terminal-sliding mode reaches zero, the convergence time can be computed with lemma 2. For the second case \( |e_m| \leq \mu_m \), the tracking error will approach zero along the general sliding manifold. Because of the small enough \( \mu_m \), the convergence time can be neglected.

**Remark 6:** The NFSTSM methods have been used in [17], [20], and other papers. This paper presents a new nonsingular fast finite-time sliding mode surface, which is different from the existing ones. The faster transient-state and higher-precision control performances are both achieved. Moreover, when sliding mode surfaces that have been presented in [17] and [20] are used to design controller. The term \( \text{diag}(|e|) \) will be composed in the reaching time; therefore, the finite reaching time could not be computed exactly. In [20], the system uncertainties and disturbance were not considered.

**Remark 7:** The control design parameters are chosen such that the system is stable and has good transient performance. With the sliding mode surface designed in (12), we choose the positive parameters \( K_m, K_{m1}, K_{m2}, K_{s1}, K_{s2}, \) and \( K_{s3} \), then the synchronization error system is stable. Next, let us analyze the transient performance. The settling time is \( T = T_1 + T_2 \) shown in (29). Based on (30), we can choose big parameters \( K_{m1}, K_{m2}, K_{s1}, K_{s2}, \) and small parameters \( K_{m1}, K_{s1}, K_{s2} \) to achieve small \( T_1 \). Based on (31), the sliding mode surface parameters \( \alpha_{m1}, \alpha_{m2}, \alpha_{s1}, \) and \( \alpha_{s2} \) are chosen to be big so as to obtain small \( T \).

**IV. Simulations**

In order to verify the effectiveness of our main results, in this section, the simulations are performed on 2-DOF manipulators
\[ M_m (q_m) \dot{q}_m + C_m (q_m, \dot{q}_m) \dot{q}_m + F_m \dot{q}_m + f_{cm} (\dot{q}_m) + g_m (q_m) + \tau_{dm} = \tau_m - F_h \]
\[ M_s (q_s) \dot{q}_s + C_s (q_s, \dot{q}_s) \dot{q}_s + F_s \dot{q}_s + f_{cs} (\dot{q}_s) + g_s (q_s) + \tau_{ds} = \tau_s + F_c \] (39)

the friction functions are as follows:
\[ F_i \dot{q}_i + f_{ci} (\dot{q}_i) = F \dot{q}_i + f_c (\dot{q}_i), \quad \text{for } i = m, s \] (40)
with
\[ F \dot{q}_i + f_c (\dot{q}_i) = \begin{cases} f_{d1} \dot{q}_i + k_1 \text{sgn}(\dot{q}_i) & \text{if } \dot{q}_i \text{ is positive} \\ f_{d2} \dot{q}_i + k_3 \text{sgn}(\dot{q}_i) & \text{if } \dot{q}_i \text{ is negative} \end{cases} \] (41)

For simulation, we choose the parameters \( m = 0.5 \) kg, \( m_1 = 1 \) kg, \( l_1 = 1 \) m, \( l_2 = 0.8 \) m, \( g = 9.81 \) m/s\(^2\), \( f_{d1} = 3 \), \( f_{d2} = 4 \), \( k_1 = 5 \), \( k_2 = 4 \), \( \tau_{dm} = \tau_{ds} = 0.3 \dot{q}_i \sin t \). The controller (21) with the parameters as \( K_{m1} = K_{s1} = \text{diag}(1, 0.8), K_{m2} = K_{s2} = \text{diag}(0.4, 0.6), \lambda_{m} = \lambda_{s} = \text{diag}(1, 1), \gamma_{m1} = \gamma_{s1} = 4, \gamma_{m2} = \gamma_{s2} = 5/7, \rho = 9/11, \alpha_{m1} = \alpha_{s1} = 10, \alpha_{m2} = \alpha_{s2} = 5, \mu_{m} = \mu_{s} = 0.0001, \) and \( T_m = T_s = 0.6 \) is adopted.

The initial states are chosen as \( q_m (0) = (0.2 \pi) \quad 0.1 \pi \dot{q}_m (0) = [0 \\ 0] \), \( q_s (0) = (0.1 \pi) \quad 0.12 \pi \dot{q}_s (0) = [0 \\ 0] \).
The initial values of the fuzzy system are chosen as \([0.01, 0.01, 0.02, 0.01, 0.02, 0.01, 0.03, 0.01]\), and the membership functions are 

\[
\mu_{A_1}(z) = e^{-\left(z - 4\right)^2}, \quad \mu_{A_2}(z) = e^{-\left(z - 2\right)^2}, \quad \mu_{A_1}(z) = e^{-z^2}, \quad \mu_{A_1}(z) = e^{-\left(z + 2\right)^2}, \quad \text{and} \quad \mu_{A_1}(z) = e^{-\left(z + 4\right)^2}.
\]

The human input force is showed in Fig. 2. To illustrate the effectiveness of the Fuzzy-NFTSM controller, some comparisons with the P+d, PD+d, and DFF will be made. And, it should be noticed that the controller parameters of P+d, PD+d, and DFF are chosen to get the same maximum control torques with the NFTSM.

First, we carried out the comparison with the commonly used P+d control approach proposed in paper [4]. The P+d control laws are given by

\[
\begin{align*}
\tau_m &= -L_m e_m - N_m \dot{q}_m + G_m \\
\tau_s &= -L_s e_s - N_s \dot{q}_s + G_s
\end{align*}
\] (42)

where \(L_m = L_s = 30\), and \(N_m = N_s = 30\). Figs. 3 and 4 illustrate the position-tracking errors at the master side and the slave side, respectively. The master and slave control torques are shown in Fig. 5. It can be seen that the slave completed the tracking successfully, and after a transient due to initial errors condition, the position-tracking errors tend to zero. Furthermore, a faster response is achieved in comparison with the P+d control approach.

After the comparison with P+d controller, the comparison with the PD+d control laws proposed in paper [3] is also conducted. The PD+d control laws are given as follows:

\[
\begin{align*}
\tau_m &= -Z_m e_m - V_m \dot{e}_m - I_m \dot{q}_m + G_m \\
\tau_s &= -Z_s e_s - V_s \dot{e}_s - I_s \dot{q}_s + G_s
\end{align*}
\] (43)

where \(Z_m = Z_s = 30\), \(V_m = V_s = 30\), and \(I_m = I_s = 30\).
Simulation results are shown in Figs. 6–8. It can be clearly seen that the proposed finite-time nonsingular fast-sliding mode controller obtains a much faster transient performance over the PD+d controller.

Moreover, the comparison with the direct force feedback (DFF) control laws proposed in paper [4] is also carried out. The DFF control laws are shown as follows:

\[
\begin{align*}
\tau_m &= -\tau_s (t - T_s) - B_m q_m \\
\tau_s &= -D_s e_s - F_s q_s
\end{align*}
\] (44)

where \(B_m = 30\), \(D_s = 30\), and \(F_s = 30\).

The simulation results are illustrated in Figs. 9–11. It can be seen that the proposed NFTSM controller also produces a much higher synchronization speed over the DFF controller.

As a result, we can conclude that the proposed finite-time nonsingular fast terminal-sliding control scheme for teleoperation system will produce a faster transient performance and higher precision synchronization performance over P+d, PD+d, and DFF control schemes.

To further illustrate the effectiveness of the control schemes proposed in this paper, the comparisons between the following two sliding mode surfaces with our NFTSM are also implemented

\[
s = \dot{e} + \alpha_2 \text{sign}(e)^2
\] (45)

\[
s = \dot{e} + \alpha_1 e + \alpha_2 \text{sign}(e)^2
\] (46)

The sliding mode surfaces (45) and (46) were proposed in [16]. We know that the finite-time sliding mode surfaces (45) and (46) will result in the singular problem. To avoid the singular problem, the switch idea used in our paper is also employed for the cases of (45) and (46). First, the comparison between the sliding mode surface (45) and our NFTSM is presented. The
simulation results are shown in Figs. 12 and 13. As we can see, the teleoperation system with NFTSM has a faster convergence speed. Next the comparison between the sliding mode surface (46) and our NFTSM is also presented. The simulation results are given in Figs. 14 and 15. The faster convergence performance also can be achieved with the NFTSM. From the simulation results, we can see the proposed method is effective and can achieve faster convergence performance.

The values of nonsingular fast sliding mode surface are shown in Fig. 16. The values of sliding mode can reach zero in finite time. And, we also can see that the reaching time is smaller than the system total convergence time. This shows the correctness of Theorem 2. In Figs. 17 and 18, the control torques of master and slave are given. The control torques for the master and the slave are bounded.

The fuzzy tuning parameters are shown in Figs. 19 and 20 for the master and slave, respectively. From these two figures, we can see that the tuning parameters converge to constants. The adaptive approximation upper bounds are shown in Figs. 21 and 22 for the master and slave, respectively. Figs. 19–22 have veri-
Fig. 18. Control torque of slave.

Fig. 19. Values of fuzzy tuning parameters at master side.

Fig. 20. Values of fuzzy tuning parameters at slave side.

Fig. 21. Upper bound at master side.

Fig. 22. Upper bound at slave side.

V. CONCLUSION

This paper addresses the control design problem for the networked teleoperation system in the presence of system uncertainties and external disturbance. To obtain faster convergence speed and higher precision performance, a new NFTSM is proposed. With the new NFTSM, a fuzzy logic-based adaptive controller, has been designed for bilateral teleoperation. The finite-time stability of the closed-loop system is proved. Moreover, the exact reaching time and the sliding time can be deduced. Comparisons are done with the P+d, PD+d, DFF, and classic TSM, NTSM controllers. The simulation results show the system with the proposed controller can get a faster convergence speed and higher convergence precision.

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YANG et al.: ADAPTIVE FUZZY FINITE-TIME COORDINATION CONTROL


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