Optimization of bus stop locations for improving transit accessibility

Steven I. Chien & Zhaoqiong Qin

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A mathematical model is developed in this paper to improve the accessibility of a bus service. To formulate the optimization model, a segment of a bus route is given, on which a number of demand entry points are distributed realistically. The objective total cost function (i.e. the sum of supplier and user costs) is minimized by optimizing the number and locations of stops, subject to non-additive users’ value of time. A numerical example is designed to demonstrate the effectiveness of the method thus developed to optimize the bus stop location problem. The sensitivity of the total cost to various parameters (e.g. value of users’ time, access speed, and demand density) and the effect of the parameters on the optimal stop locations are analyzed and discussed.

**Keywords:** Bus, Stop, Transit, Planning, Accessibility, Cost, Optimization

**1. INTRODUCTION**

The density and locations of bus stops are usually used to determine the accessibility of bus services, which significantly influences transit system performance and level of service. In a newly developed suburban region where a bus route is nearby, adding a number of stops onto the bus route can stimulate ridership because of reduced access time. However, the corresponding user in-vehicle time and supplier cost might increase due to the excess acceleration and deceleration delays incurred by buses serving the additional stops.

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*Corresponding author. E-mail: chien@adm.njit.edu*
A number of previous studies [1—6] have focussed on planning bus transit systems while considering the locations of bus stops. However, demand was considered to be distributed continuously along an existing transit route. The methods applied for analyzing transit system performance were based on the relationships between stop spacing, service headway, fleet size and round trip travel time. The objectives of those studies were using these relationships to evaluate system capacity versus actual demand and required headway versus policy headway to minimize the total cost (the sum of supplier and user costs). It was believed that the performance of a transit system could be significantly improved if the spacing of bus stops was optimized.

The review of past research for this study on optimizing locations of bus stops can be classified into three categories. The first involved studies that focused on locating bus stops based on simplified demand distribution. A survey on bus stop design was conducted by Fitzpatrick et al. [4] who developed guidelines for allocating bus stops considering bus patron’s convenience, safety and access time, as well as the efficiency of transit operations. Based on given traveler preferences and budget constraints from the transit provider, Van Nes and Bovey [7] developed an analytical model that optimized stop spacing and line spacing, while a number of performance indices (i.e. travel time, operator costs, and patronage) were analyzed. Due to the fact that buses compete for right of way with other transportation modes, Brouwer [1] and Wirasinghe and Ghoneim [6] optimized the locations of bus stops considering the costs incurred not only by bus passengers and transit suppliers, but also by other road users, local residents and businesses.

The second category focused on joint optimization of bus route and stop spacings for a bus system. Holroyd [8] analyzed a grid bus network with a given origin–destination (O–D) demand that was uniformly distributed over an infinite plane. The route spacing and headway were optimized, which achieved the minimum total cost. Later Chien and Schonfeld [9] optimized a grid transit system in a heterogeneous urban area without oversimplifying the spatial and demand characteristics. An iterative algorithm was developed to minimize the total cost, while the bus route spacing, stop spacing and headways were optimized. Chien and Zang [10] and Chien et al. [11] found the optimal number and locations of bus routes serving a CBD and a residential area by minimizing the total cost using genetic algorithms. Since passengers were assumed to be able to access buses anywhere on the route (e.g. jitney service), the locations of the bus stops were not considered.
In the third category, several studies have considered temporal demand while optimizing a bus system. Gerrard and Hundle [12] optimized a service area by minimizing the total cost considering time varying demand. Bramel and Simchi-Levi [13] applied heuristic methods to solve a vehicle routing problem in which customers requested service with a certain probability during a given time period. Most of the aforementioned studies considered additive time cost while optimizing their objective functions. However, in the real world the cost of time is rarely linear. Thus, Chien et al. [14] evaluated various feeder bus systems considering probabilistic time-varying demand and non-additive time costs.

In this paper, a mathematical model is formulated to optimize the number and locations of bus stops that achieve the minimum total cost. A realistic demand distribution based on a general street configuration is considered (Fig. 1). The concept of non-additive time cost is also applied in the development of the model.

2. MODEL FORMULATION

The objective of this paper is to develop a mathematical model to optimize the number and locations of bus stops. A segment of a bus route is assumed in which a number of entry points associated with different boarding and alighting demand is considered. The following assumptions are made to formulate the objective total cost function:
Passengers always walk to the nearest stop. Thus, access time can be estimated from the ratio of access distance and speed. Buses will serve all stops, while the acceleration and deceleration rates are fixed. Therefore, the acceleration and deceleration delays at all stops are assumed to be identical.

The demand originating at entry point \( i \) in direction \( d \), called \( q_{id} \), and destined for entry point \( i \) in direction \( d \), called \( d_{id} \), are known. The boarding and alighting demand of an entry point is uniformly distributed over a period of time, but the volume may differ from other entry points.

Bus headway is given and will not be altered in this study. Thus, sufficient fleet size is assumed to cover extended round trip travel time due to serving the newly added stops. The average passenger wait time is assumed to be half the headway.

Demands in both service directions are assumed to be even. Thus, \( q_{id} \) in direction 1 is equal to \( d_{id} \) in direction 2. The service capacity is always sufficient to satisfy the demand. This assumption can be relaxed but the optimal number and locations of stops may be different for both service directions.

The user cost is a nonlinear function of time. In this study, a quadratic function of time is assumed to estimate the user cost. For example, if the user travel time is \( t \), the user cost would be \( t^2 \) multiplied by the value of time.

The demand for the entry point of the segment studied includes boarding demand, \( q_{id} \), and alighting demand, \( d_{id} \). In order to improve the accessibility of the bus service shown in Fig. 1, a number of stops should be appropriately located along the segment. Since buses will serve all stops, the increased bus travel time per trip will increase proportionally with the number of stops. For each stop \( j \), the increased travel time consists of two parts: (1) acceleration/deceleration delay; and (2) dwell time.

### 2.1. Acceleration/Deceleration Delay

In order to estimate acceleration/deceleration delay, \( t_{j,1} \), at a stop, Eq. (1) derived by Chien [15] is applied. Thus:

\[
t_{j} = \frac{v}{2a} + \frac{v}{2b}
\]  

(1)
where \( v \), \( a \), and \( b \) represent average speed, acceleration rate, and deceleration rate, respectively. The acceleration/deceleration delays at all stops are assumed identical as discussed in Assumption 1.

### 2.2. Dwell Time

The duration of bus dwell time, \( t_2 \), at a stop depends on the number of passengers and average boarding/alighting time per passenger, \( w \). Thus:

\[
t_2 = Qw
\]

where \( Q \) represents the number of passengers boarding and alighting at a stop. As \( Q \) varies at different places and time periods, the dwell time may differ over space and time as well.

Given that the bus headway is \( h \) hours, the total number of boarding and alighting passengers, \( A \), served by a bus at all new stops on the segment is

\[
A = h \sum_{i=1}^{n} (q_{id} + d_{id})
\]

where \( n \) and \( d \) represent the number of demand entry points and the index of service direction, respectively. If there are \( m \) bus stops located on the segment, the increased traveling time due to the total acceleration/deceleration delay, \( T_1 \), and dwell time, \( T_2 \), can be obtained from Eqs (4) and (5), respectively.

\[
T_1 = mt_1
\]

\[
T_2 = Aw
\]

### 2.3. Total Cost Function

The total cost function defined here is the sum of the increased supplier and user costs, while the user cost includes the access, wait and in-vehicle costs.

#### 2.3.1. Supplier Cost, \( C_s \)

The supplier cost is the product of fleet size, \( F \), and bus operating cost, \( u_c \). Thus:

\[
C_s = u_c F
\]
The fleet size can be obtained by the round travel time divided by the headway. Since the increased round-trip travel time for the bus serving \( m \) stops is \( 2(T_1 + T_2) \), the increased \( F \) required for serving the new stops can be obtained from the increased round-trip time divided by the headway as formulated in Eq. (7):

\[
F = \frac{2(T_1 + T_2)}{h}
\]  

(7)

Based on Eq. (7), a fractional fleet size may be produced, which will be rounded up to accommodate the fixed service headway, \( h \).

2.3.1. User Cost, \( C_u \)

The user cost is the sum of three cost components:

\[ C_u = C_x + C_w + C_v \]  

(8)

where \( C_x \), \( C_w \) and \( C_v \) represent access, wait and in-vehicle costs, respectively. As the headway is given and demand is uniformly distributed, the average passenger wait time is a half of the headway. Thus, the user wait cost \( C_w \) is constant and will not affect the optimal solution. Therefore, the user cost can be treated as the sum of \( C_x \) and \( C_v \):

\[ C_u = C_x + C_v \]  

(9)

Since the user access and in-vehicle costs are considered as a non-linear function of time, the user access cost, \( C_x \), and in-vehicle cost, \( C_v \), per passenger can be estimated through Eqs (10) and (11), respectively. Thus:

\[
c_x = u_x f(t_{x}) = u_x t_{xi}^2
\]  

(10)

\[
c_v = u_v f(t_{v}) = u_v t_{vi}^2
\]  

(11)

where \( u_x \) and \( u_v \) stand for the values of users’ access and in-vehicle time, while \( t_{x} \) and \( t_{v} \) represent the access and in-vehicle times of passengers at entry point \( i \), respectively. Thus, the user access cost, \( C_x \), can be derived as:

\[
C_x = 2u_x \sum_{i=1}^{n} [(q_{id} + d_{id})t_{xi}^2]
\]  

(12)

where \( 2(q_{id} + d_{id}) \) means the hourly round-trip boarding and alighting demand at entry point \( i \) in direction \( d \). Since passengers will access
the nearest stop, the access time, \( t_{ai} \), from entry point \( i \) to the nearest stop can be formulated as:

\[
 t_{ai} = \min \left( \frac{|l_{i-1} - s_1|}{g}, \ldots, \frac{|l_{i-1} - s_j|}{g}, \ldots, \frac{|l_{i-1} - s_m|}{g} \right)
\]  

(13) 

where \( g \) is the average passenger access speed. As shown in Fig. 1, \( l_{i-1} \) is the distance from entry point \( i \) to the first entry point, and is the distance from bus stop \( j \) to the first entry point.

The user in-vehicle cost \( C_v \) includes two components, \( C_{v1} \) and \( C_{v2} \). \( C_{v1} \) is incurred by flow \( S_d \) through the segment in direction \( d \) and can be obtained from the product of \( S_d \), value of in-vehicle time \( u_v \), and the weighted increased round-trip traveling times \( 2(T_1 + T_2) \). Note that the through flow \( S_d \) is determined before the new stops are included into the service. Thus:

\[
 C_{v1} = 2u_v S_d(T_1 + T_2)^2
\]  

(14) 

\( C_{v2} \) is incurred by the boarding and alighting demand traveling along the segment of the bus route. The segment with \( n \) entry points can be divided into \((m + 1)\) links by \( m \) stops, where link 1 serves demand from the first entry point to the first bus stop and vice versa. For \( 2 \leq p \leq m \), link \( p \) connects stops \((p - 1)\) and \( p \). The last link \((m + 1)\) connects stop \( m \) and the last entry point.

On link 1, only alighting demand, \( a_1 \), at stop 1 and through demand, \( S_d + A S_d \), are traversing. Thus:

\[
 A S_d + a_1 = \sum_{i=1}^{n} d_{id}
\]  

(15) 

where \( A S_d \) represents the new generated demand by the new stops.

The average running time and deceleration delay on link 1 are \( x_1/v \) and \( v/2b \), respectively, where \( x_i \) (\( = s_1 \)) represents the distance from the first entry point to the first bus stop and \( x_i = s_i - s_{i-1} \) for \( i \geq 2 \). Therefore, the bus travel time on link 1 \( (t_{i1}) \) is

\[
 t_{i1} = \frac{x_1}{v} + \frac{v}{2b}
\]  

(16) 

where the acceleration delay at the first stop is not counted as travel time on link 1. The round trip user in-vehicle cost occurring on link 1 \( (C_{v21}) \) is the product of the value of in-vehicle time, \( u_v \), increased
demand per hour, \( \Delta S_d + a_i \), and the weighted round-trip travel time on link 1 that is \( 2t_{1i} \). Thus:

\[
C_{21} = 2u_i(\Delta S_d + a_i)t_{1i}^2
\]

(17)

Assume that \( I_p \) stands for the number of entry points of link \( p \), where \( 2 \leq p \leq m \). Thus, the through demand, \( a_p \), of link \( p \) is the sum of the through demand on link 1, \( a_i \), and the accumulated difference of boarding and alighting demand:

\[
a_p = (\Delta S_d + a_i) + \sum_{i=1}^{I_p} (q_{id} - d_{id})
\]

(18)

Link \( p \) connects stops \( (p-1) \) and \( p \) where buses accelerate from stop \( (p-1) \), travel with constant speed, and then decelerate and arrive at stop \( p \). From Eq. (1), the bus travel time on link \( p \) can be derived as:

\[
t_{ip} = \frac{x_p}{v} + \left( \frac{v}{2a} + \frac{v}{2b} \right)
\]

(19)

where \( x_p \) the distance from stop \( (p-1) \) to stop \( p \). Thus, the round trip user in-vehicle cost \( (C_{22}) \) occurring on links 2 through \( m \) is:

\[
C_{22} = 2u_i \sum_{p=2}^{m} (a_p t_{ip}^2)
\]

(20)

Similarly, the increased demand, \( a_{m+1} \), on link \( m+1 \) can be formulated as Eq. (21):

\[
a_{m+1} = \sum_{i=1}^{n} q_{id}
\]

(21)

Note that the length of link \( m+1 \) is

\[
\left( L - \sum_{p=1}^{m} x_p \right)
\]

which connects stop \( m \) and entry point \( n \). Thus, the bus traveling time \( (t_{nm+1}) \) on that link can be derived as:

\[
t_{nm+1} = \frac{L - \sum_{p=1}^{m} x_p}{v} + \frac{v}{2a}
\]

(22)
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where the deceleration delay $v/2b$ at stop $m$ is not a part of the travel time on link $m+1$. Thus, the user round trip in-vehicle cost, $C_{v23}$, occurring on that link can be formulated as:

$$C_{v23} = 2u_o a_m + r^2_{m+1}$$

The user round trip in-vehicle cost, $C_v$, can be derived as the sum of $C_{v1}$ and $C_{v2}$. $C_{v1}$ can be obtained from Eq. (14), while $C_{v2}$ is the sum of $C_{v21}$, $C_{v22}$ and $C_{v23}$ as formulated in Eqs (17), (20) and (23), respectively. The total cost can thus be estimated by the sum of $C_o$, $C_v$ and $C_x$, within which the decision variables include an integer variable $m$ and a number of continuous variables $S_p$ to determine the optimal locations of the bus stops.

3. OPTIMIZATION ALGORITHM

The objective function formulated in this study is a mixed integer, quadratic and multi-dimensional function. It contains an integer variable $m$ and a number of continuous variables $\{s_1, s_2, \ldots, s_m\}$. Since $m$ is a finite number, the optimal results can be obtained numerically. The general procedure to solve this optimization problem is stated as follows and shown in Fig. 2:

Step 1: Start with $m = 1$. If $m < n$, go to Step 2; If $m = n$, go to Step 3
In this step, the value of $n$ (the number of entry points) is given, and $m$ is enumerated from 1 to $n$.

Step 2: Determine locations of stops
There are in total $N = \binom{n-1}{m}$ combinations for $m$ stops and $n$ entry points, where

$$\binom{n}{m} = \frac{(n-1)!}{m!(n-1-m)!}$$

If $m < n$, there is at most one stop between two adjacent entry points. If $m = n$, the locations of the stops will be exactly at the entry points.

Step 3: Calculate the operator cost $C_o$ with Eq. (7)
In this step, the inputs include bus operating cost, $u_o$, bus headway, $h$, boarding and alighting demands, $q_{ld}$ and $q_{ld}$, and locations of all entry points and through flow, $S_d$ and $AS_d$.

Step 4: Calculate the user access cost $C_x$ with Eq. (12)
In this step, the inputs include the value of users’ access time,
FIGURE 2 Optimization algorithm

Step 5: Calculate the user in-vehicle cost $C_u$ with Eqs (14), (17), (20) and (23)
In this step, the inputs include the values of in-vehicle time, $u_v$, acceleration speed, $a$, deceleration speed, $b$, the length of the bus route segment, $L$, and bus average speed, $v$.

Step 6: Calculate the total cost $C_T$
Calculate $C_T$ and search the minimal $C_T$ when all combinations are enumerated.

$u_v$, passenger access speed, $g$ and the distances $l_{i-1}$ from the first entry point to all entry points.
Step 7: Update the minimum total cost $C_T^*$
If $m = 1$ and $k = 1$, $C_T^* = \text{Min}(C_T)$. Otherwise, if $\text{Min}(C_T) < C_T^*$, $C_T^* = \text{Min}(C_T)$.

Step 8: Find the optimal solution.
When $m$ is enumerated from 1 to $n$, the minimum cost $C_T^*$ associated with the optimal $m^*$ variables and $\{s_1, s_2, \ldots, s_m\}^*$ can be obtained.

4. NUMERICAL EXAMPLE

The main purpose of this section is to demonstrate the model developed above that can optimize the number and locations of bus stops along a three-mile long segment of a bus route as shown in Fig. 3. Assume that there is a newly developed residential area where demand can access a bus service from four entry points. The lengths between the successive entry points are 1, 1.2 and 0.8 miles, respectively. A number of new stops can be located to improve the service accessibility. The algorithm developed in the previous section is used to optimize stops along the segment. Tables I and II define all parameters and their baseline values that are used in the numerical analysis. The minimum total cost is 138.8 US$/h that is achieved by locating three stops at 0.03, 1.02 and 2.63 miles away from the first entry points, respectively (see Table III).

Based on the optimal number and locations of stops, the average access distance per passenger is 0.19 miles. Thus, the average access time is 4.56 min based on the average passenger access speed of 2.5 mph. According to Eq. (3), the total number of passengers from the segment is 20 passengers per bus, and the resulting dwell time, $T_2$ is 0.01 h. In addition, the increased travel time incurred by vehicle acceleration and deceleration delay, $T_1$ is 0.045 h. Therefore, the excess bus round trip travel time due to serving passengers at the three
### Table I. Variable definitions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
<th>Baseline Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Vehicle acceleration rate</td>
<td>mi/s$^2$</td>
<td>0.12</td>
</tr>
<tr>
<td>$a_p$</td>
<td>Boarding and alighting demand on link $p$</td>
<td>pass/h</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>Boarding/alighting passengers from new stops</td>
<td>pass/bus</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>Vehicle deceleration rate</td>
<td>mi/s$^2$</td>
<td>0.12</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Supplier cost</td>
<td>US$/h</td>
<td></td>
</tr>
<tr>
<td>$c_r$</td>
<td>Total cost</td>
<td>US$/h</td>
<td></td>
</tr>
<tr>
<td>$c_u$</td>
<td>User cost</td>
<td>US$/h</td>
<td></td>
</tr>
<tr>
<td>$c_v$</td>
<td>User in-vehicle cost</td>
<td>US$/h</td>
<td></td>
</tr>
<tr>
<td>$c_w$</td>
<td>User access cost</td>
<td>US$/h</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>Index of traffic direction</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$d_{id}$</td>
<td>Alighting demand in direction $d$ at entry point $i$</td>
<td>pass/h</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>Increased fleet size</td>
<td>vehicles</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>Passenger walking speed</td>
<td>mph</td>
<td>2.5</td>
</tr>
<tr>
<td>$h$</td>
<td>Bus headway</td>
<td>h</td>
<td>0.25</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of entry points</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Number of entry points on link $p$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$j$</td>
<td>Index of bus stops</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$l_{i1}$</td>
<td>Distance from entry point $i$ to the first entry point</td>
<td>miles</td>
<td>–</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the study segment</td>
<td>miles</td>
<td>3.0</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of new bus stops</td>
<td>stops</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Number of entry points</td>
<td>entries</td>
<td>4</td>
</tr>
<tr>
<td>$n_s$</td>
<td>Number of combinations of stop locations</td>
<td>–</td>
<td>$c^i$</td>
</tr>
<tr>
<td>$p$</td>
<td>Index of links</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$q_{id}$</td>
<td>Boarding demand in direction $d$ at entry point $i$</td>
<td>pass/h</td>
<td></td>
</tr>
<tr>
<td>$Q_j$</td>
<td>Number of boarding/alighting passengers at stop $j$</td>
<td>pass/trip</td>
<td></td>
</tr>
<tr>
<td>$s_j$</td>
<td>Distance from stop $j$ to the first entry point</td>
<td>miles</td>
<td>–</td>
</tr>
<tr>
<td>$S_{jd}$</td>
<td>Through flow in direction $d$</td>
<td>pass/h</td>
<td>120</td>
</tr>
<tr>
<td>$AS_d$</td>
<td>Through flow attracted by new stops in direction $d$</td>
<td>pass/h</td>
<td>–</td>
</tr>
<tr>
<td>$t_1$</td>
<td>Acceleration/deceleration delay time at a stop</td>
<td>h</td>
<td>–</td>
</tr>
<tr>
<td>$t_2$</td>
<td>Dwell time at a stop</td>
<td>h</td>
<td>–</td>
</tr>
<tr>
<td>$t_{i1}$</td>
<td>Access time per passenger at entry point $i$</td>
<td>h</td>
<td>–</td>
</tr>
<tr>
<td>$t_{i2}$</td>
<td>In-vehicle time per passenger at entry point $i$</td>
<td>h</td>
<td>–</td>
</tr>
<tr>
<td>$t_{ip}$</td>
<td>Travel time on link $p$</td>
<td>h</td>
<td>–</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Total acceleration/deceleration delay</td>
<td>h</td>
<td>–</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Total dwell time</td>
<td>h</td>
<td>–</td>
</tr>
<tr>
<td>$u_o$</td>
<td>Bus operating cost</td>
<td>US$/veh-h</td>
<td>50</td>
</tr>
<tr>
<td>$u_a$</td>
<td>Value of users’ access time</td>
<td>US$/pass-h</td>
<td>20</td>
</tr>
<tr>
<td>$u_v$</td>
<td>Value of users’ in-vehicle time</td>
<td>US$/pass-h</td>
<td>12</td>
</tr>
<tr>
<td>$v$</td>
<td>Average vehicle operating speed</td>
<td>mph</td>
<td>15</td>
</tr>
<tr>
<td>$x_p$</td>
<td>Length of link $p$</td>
<td>miles</td>
<td>–</td>
</tr>
<tr>
<td>$w$</td>
<td>Average boarding &amp; alighting time per passenger</td>
<td>h</td>
<td>1/1800</td>
</tr>
</tbody>
</table>
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TABLE II. Demand (pass/h) distribution for all entry points

<table>
<thead>
<tr>
<th>Entry point (i)</th>
<th>( q_{id} )</th>
<th>( d_{id} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

TABLE III. Total costs for various number and locations of stops

<table>
<thead>
<tr>
<th>No. of stops</th>
<th>Distance (miles)</th>
<th>Total cost (US$/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stop 1</td>
<td>Stop 2</td>
</tr>
<tr>
<td>1</td>
<td>1.00</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>1.51</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>2.20</td>
<td>2.20</td>
</tr>
<tr>
<td>4</td>
<td>0.41</td>
<td>1.95</td>
</tr>
<tr>
<td>5</td>
<td>1.40</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>2.46</td>
</tr>
<tr>
<td>7</td>
<td>0.34</td>
<td>2.56</td>
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<tr>
<td>8</td>
<td>0.60</td>
<td>2.18</td>
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<tr>
<td>9</td>
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<td>1.46</td>
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<tr>
<td>10</td>
<td>0.03</td>
<td>1.02</td>
</tr>
<tr>
<td>11</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Distance means the length between the stop location and the first entry point.

5. SENSITIVITY ANALYSIS

The sensitivity analysis is conducted to show the relationship among various parameters and design variables. The value of each parameter is changed based on their baseline values and the new optimal decision variables are computed. The results are shown in Figs 4–7. Fig. 4 shows the minimum total costs while altering the number of stops for the different values of users’ in-vehicle time, \( u_v \). When \( u_v \) increases, the minimum total cost increases. It is worth noting that as decreases to 3 US/pass-h, the optimal number of stops increases from 3 to 4.

Figure 5 shows the minimum total costs for the different values of users’ access time, \( u_a \). Similar to the impact of \( u_v \) on the total cost, the
minimum total cost increases while $u_v$ increases. Fig. 6 shows the minimum total costs for the different values of passenger access speeds. It is found that as the passenger access speed reduces, the optimal number of stops tends to increase. Thus, if passengers can use a personal transportation mode (e.g. bicycle or scooter), the number of stops as well as the total cost can be reduced. Fig. 7 shows the
minimum total costs versus the number of stops for various demand multipliers. It is found that the optimal number of stops decreases from 3 to 2 as the demand multiplier decreases from 1.5 to 0.01.

6. CONCLUSION

The development of an analytical model in this study has been demonstrated, while the relationship between parameters (e.g. access speeds (g))

FIGURE 6 Total cost plotted against the number of stops for various passenger access speeds (g)

FIGURE 7 Total cost plotted against the number of stops for various demand multipliers
speed, values of users’ time, bus operating cost, and etc.) and variables (the number and locations of bus stops) has been derived. The optimal results have achieved a cost-effective solution for the segment with \( n \) entry points. The model as developed can be applied to determine the optimal number and locations of bus stops considering realistic street patterns and spatial boarding/alighting demand distributions. It is found that the optimal number and locations of the stops will be affected by users’ value of time, access speed and demand.

In order to advance the applicability of the model, some conditions assumed in this study may be relaxed. Considering demand elasticity and the effect of dwell time on users’ in-vehicle cost into the objective function would be desirable extensions. Additional fleet size due to time-varying demand can be taken into account and lead toward a more comprehensive analysis. However, this may increase the complexity of the objective function.

**References**
